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Self-dual Morphological Methods Using Tree Representation

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Outline

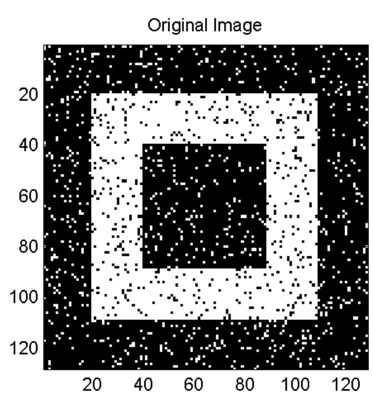
- This work introduces a general framework for tree-based morphological processing
  - Heart of the scheme: A complete inf-semilattice (CISL) on a tree-representation domain
  - Given a tree, we show how to use this CISL to process images
- Particular case
  - Extreme-watershed tree
  - Applications shown

Trees in Image Processing

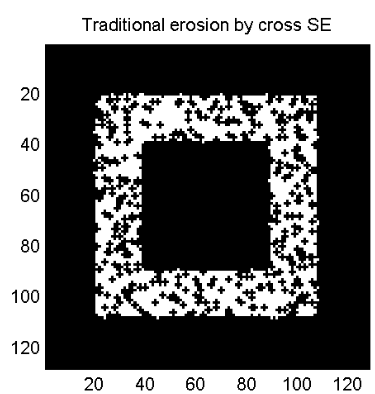
- A common use of trees:
  - Represent flat zones of a given image by a tree scheme
  - Prune tree branches
  - Reconstruct the image
  - Result: Connected filtering
- A couple of examples:
  - Binary partition trees (P. Salambier and L. Garrido, 2000)
  - Tree of Shapes (P. Monasse and F. Guichard, 2000)

Self-Duality

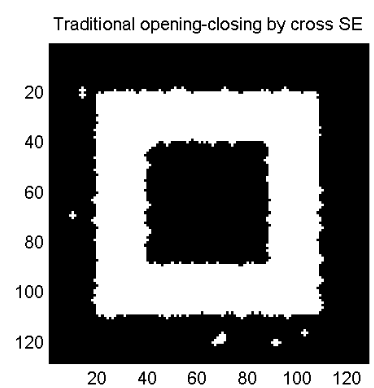
- Operator ψ is self-dual, if $\psi(f)=-\psi(-f)$.
  - Same treatment of dark and light objects.
  - Important to many applications, including filtering.
- Linear operators are self-dual; Morphological operators are usually not.
- Many trees are self-dual and yield self-dual morphological filters



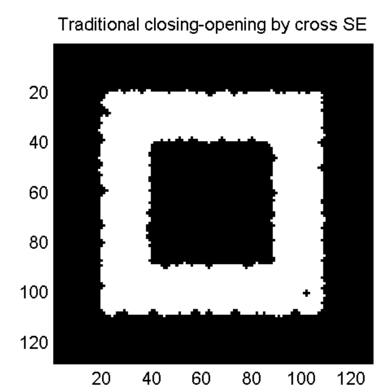
Original Image



Erosion by cross SE



Close-open

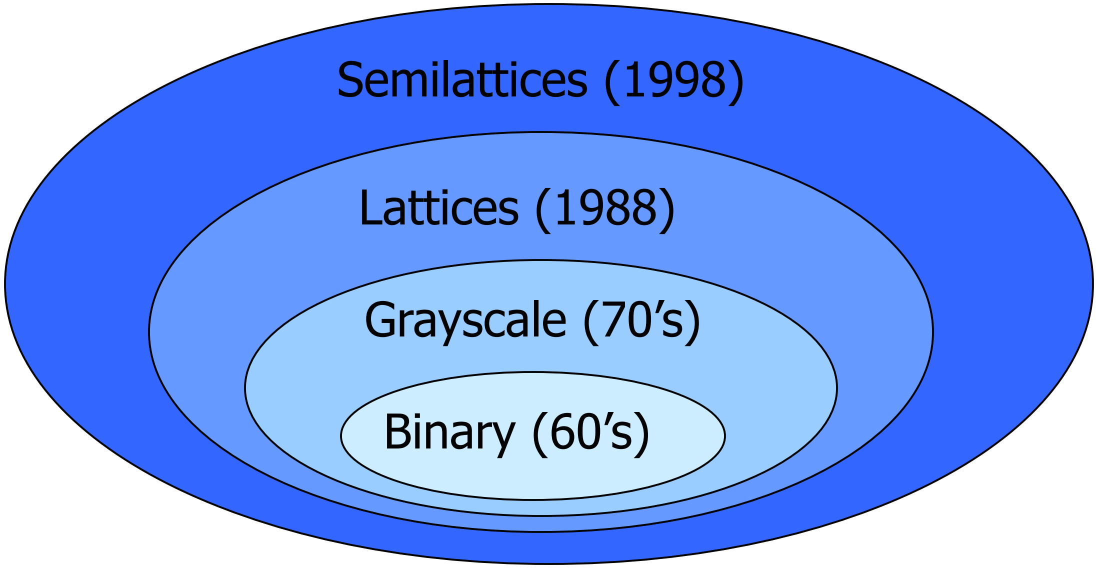


Open-close

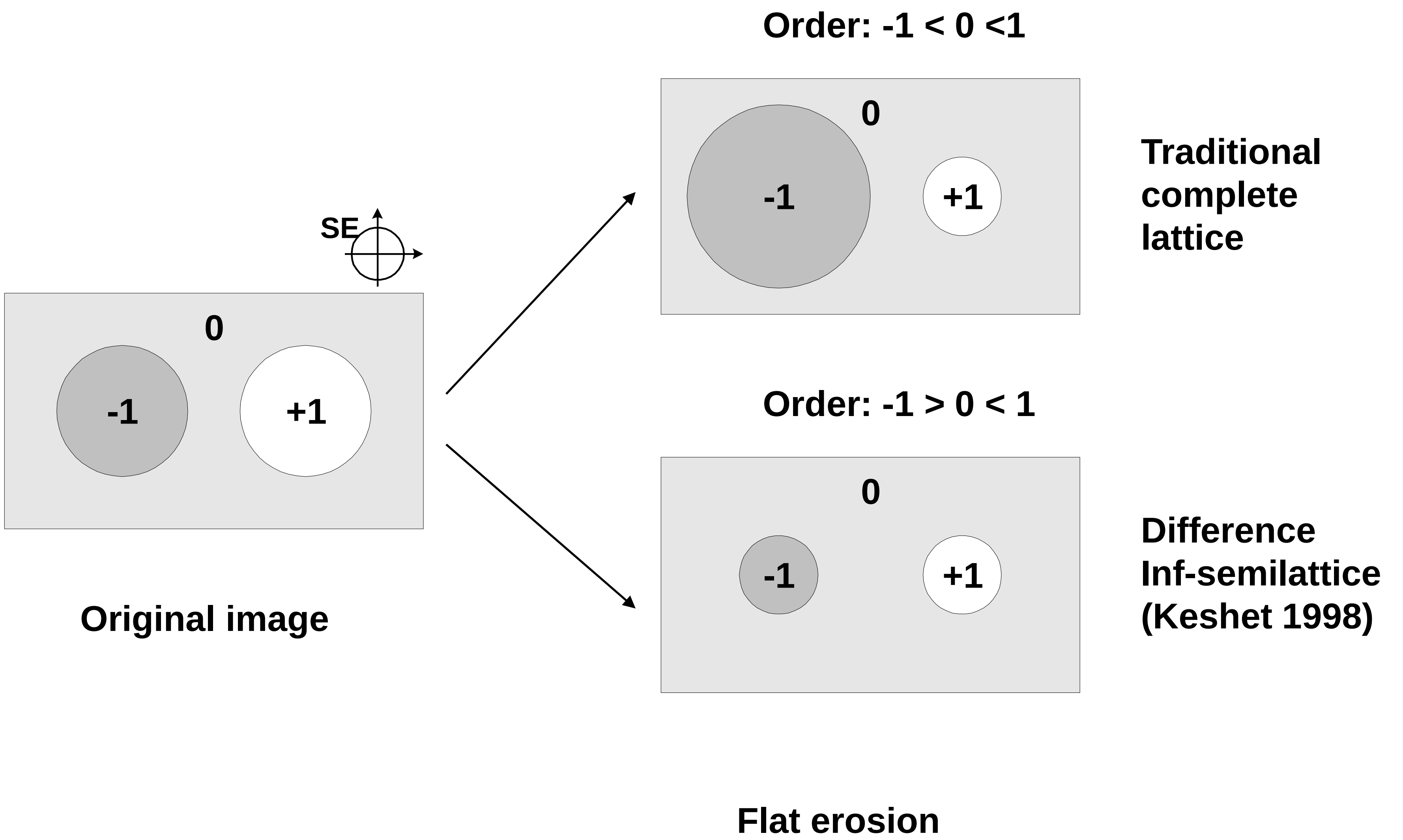
The Problem

- Trees do not yield morphological adjunctions, openings, closings, etc.
  - Usually connected filters
- Size criterion can not be implemented
- How to obtain self-dual adjunctions, using trees?

Morphology on Semilattices

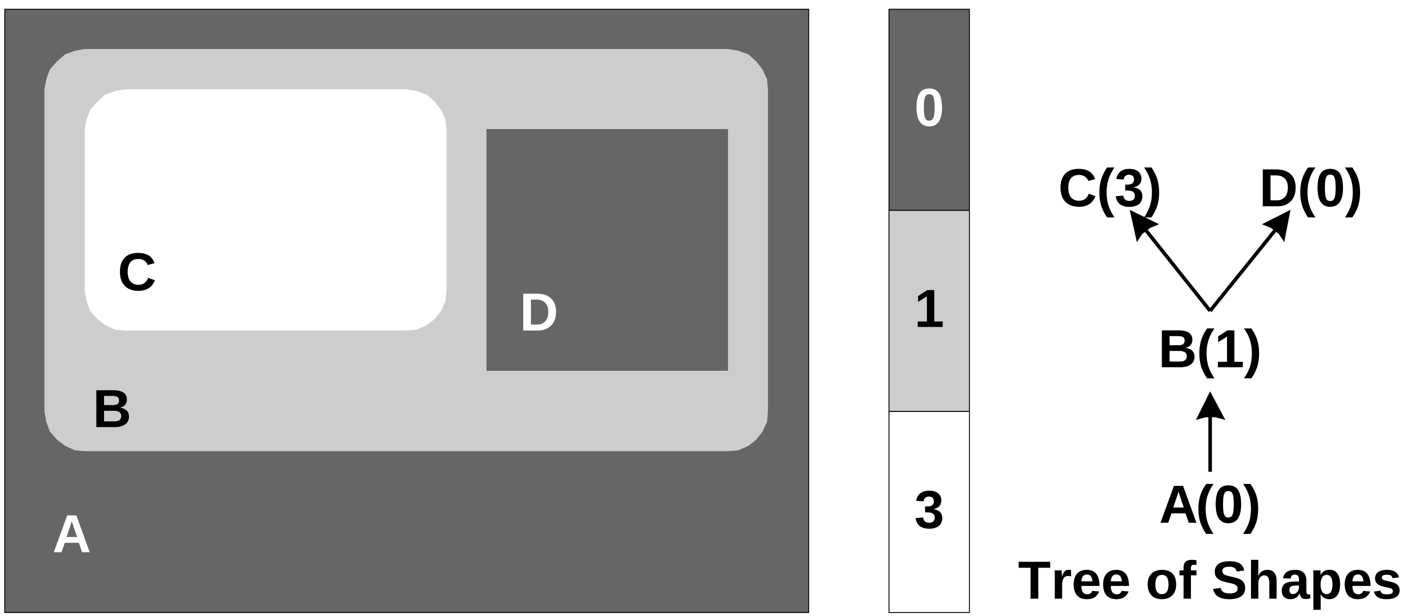


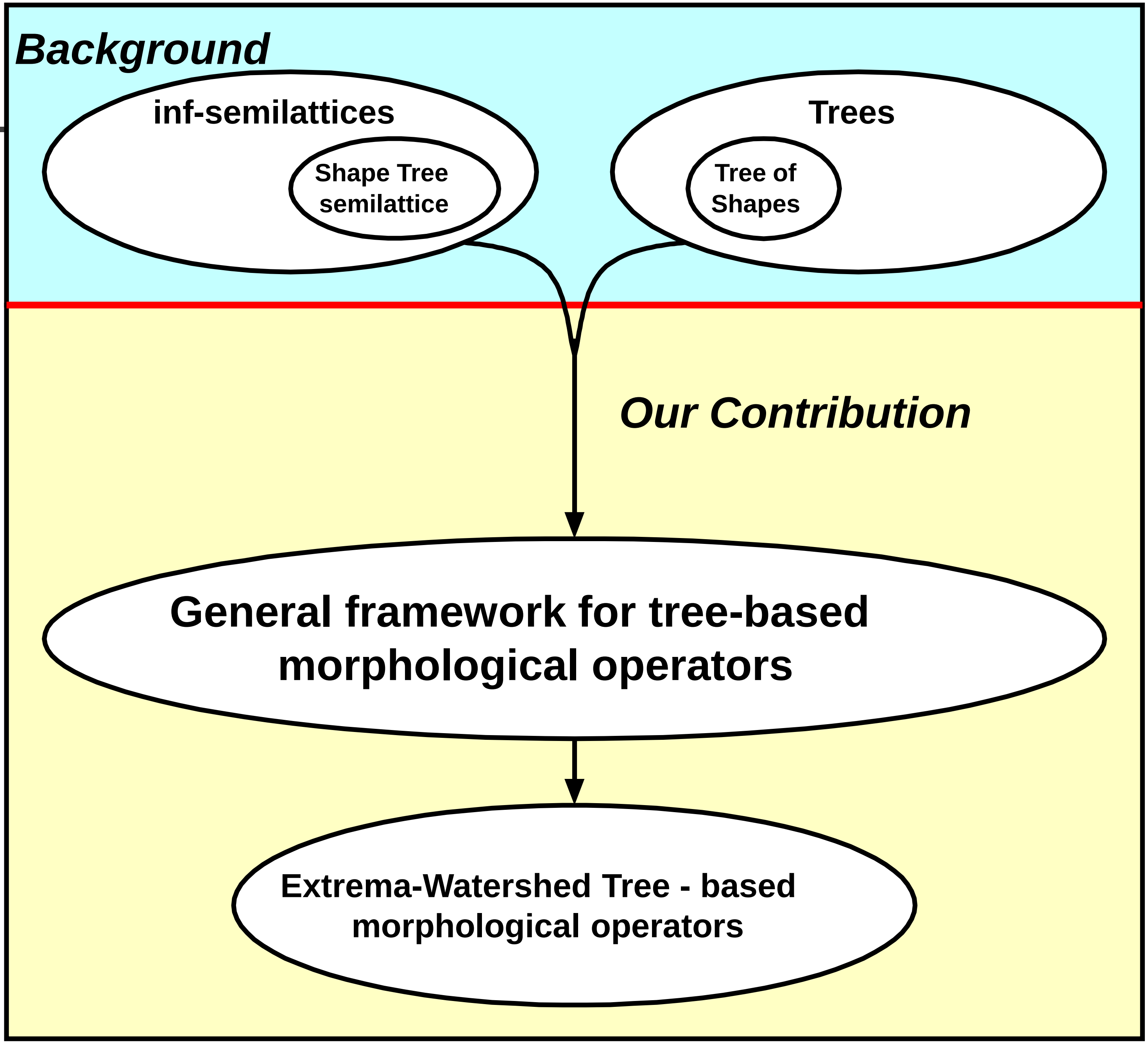
Self-duality on inf-semilattices



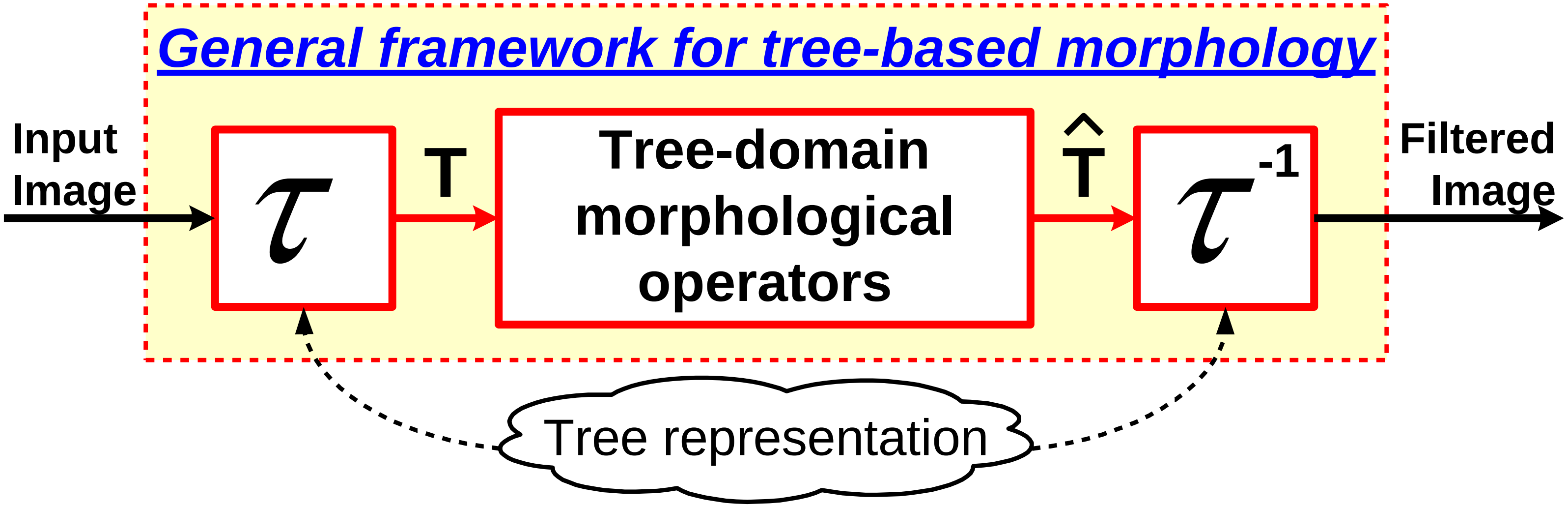
Shape-tree semilattice (Keshet, 2005)

- Uses the datum of the tree of shapes to define a complete inf-semilattice
- Self-dual morphological operators result



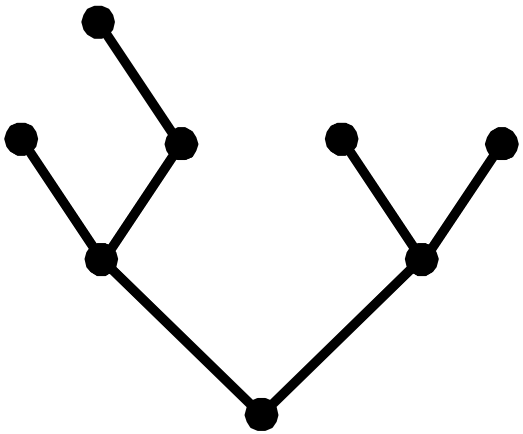

Background
inf-semilattices
Shape Tree semilattice
Trees
Tree of Shapes
Our Contribution
General framework for tree-based morphological operators
Extrema-Watershed Tree - based morphological operators

Tree-based morphology



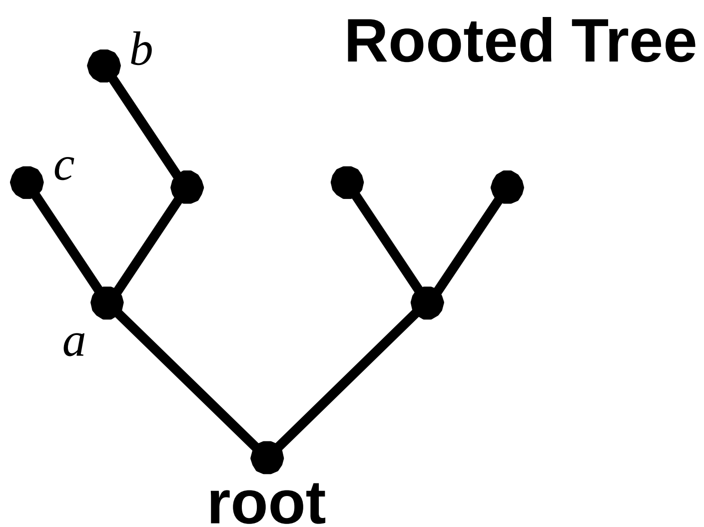
Trees

- A tree t = (V, E) is a connected graph with no loops, where V is a set of vertices and E a set of edges connecting vertices.
- A rooted tree is a tree with a root vertex.



Natural order on rooted trees

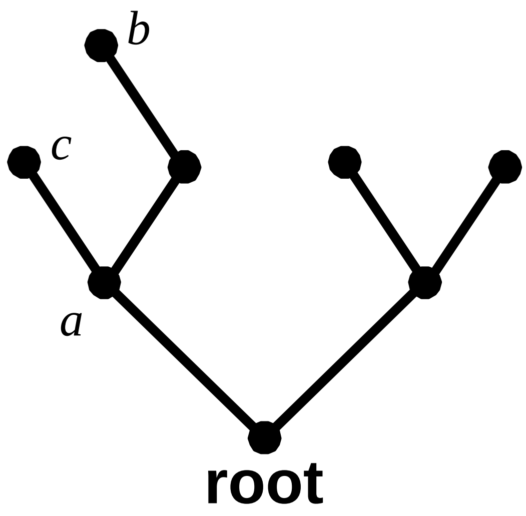
- On a rooted tree, there is a natural partial order:
  - A vertex a is smaller than b if a is in the (unique) path connecting b to the root



Natural order on rooted trees

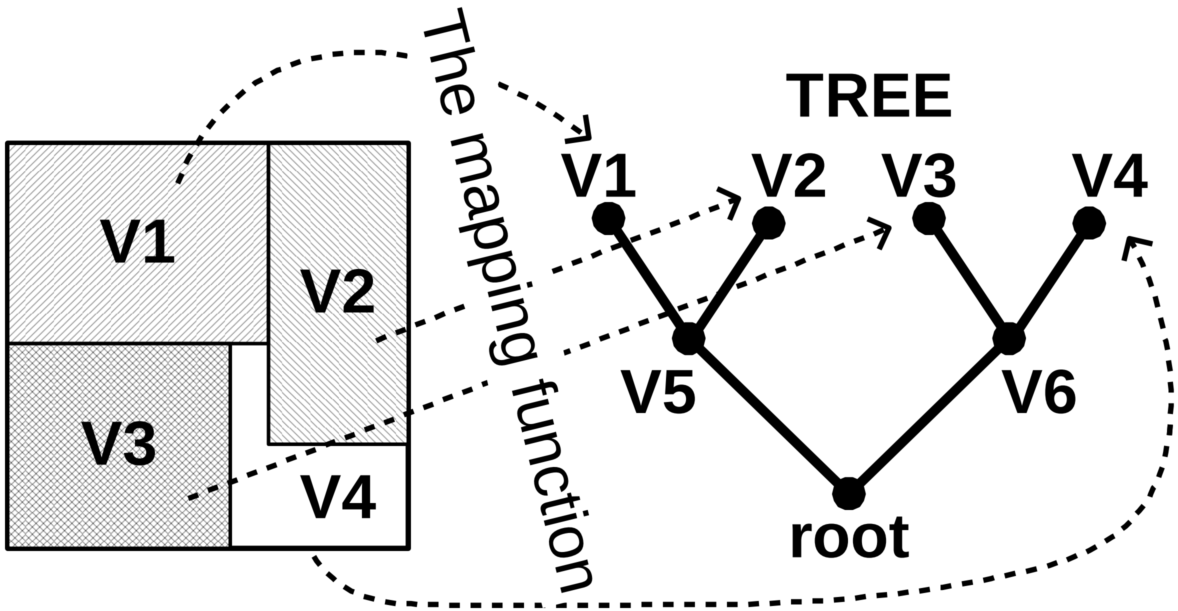
- This order turns the set of vertices into a complete inf-semilattice
  - The least element is the root
  - The infimum is the common ancestor (e.g., a is the infimum of $\{b,c\}$)

Rooted Tree



Tree representation

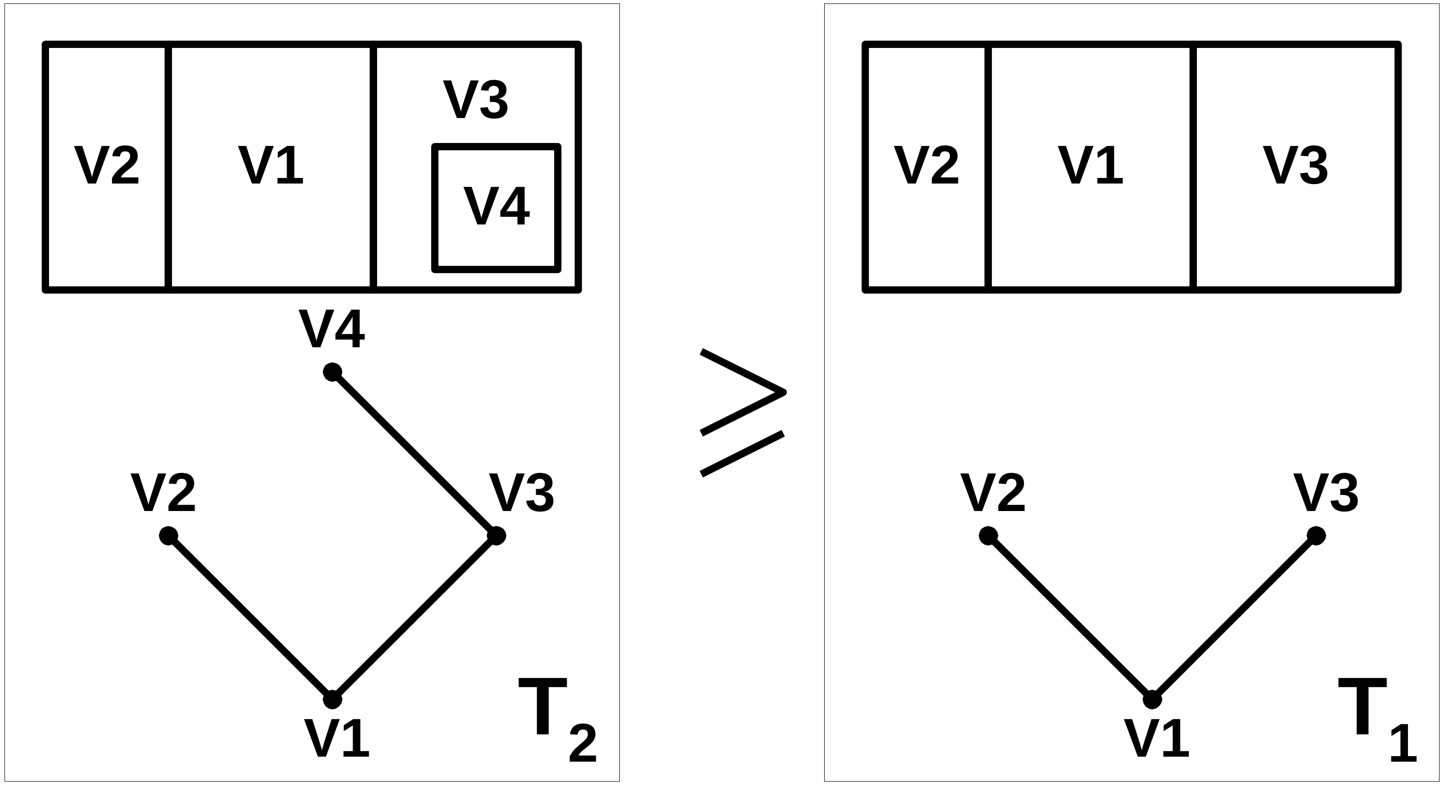
- Tree : t = (V, E)
- Mapping function : M : $\mathbf{R}^2 \rightarrow V$
- Tree representation : T = (t, M)



The tree representation order

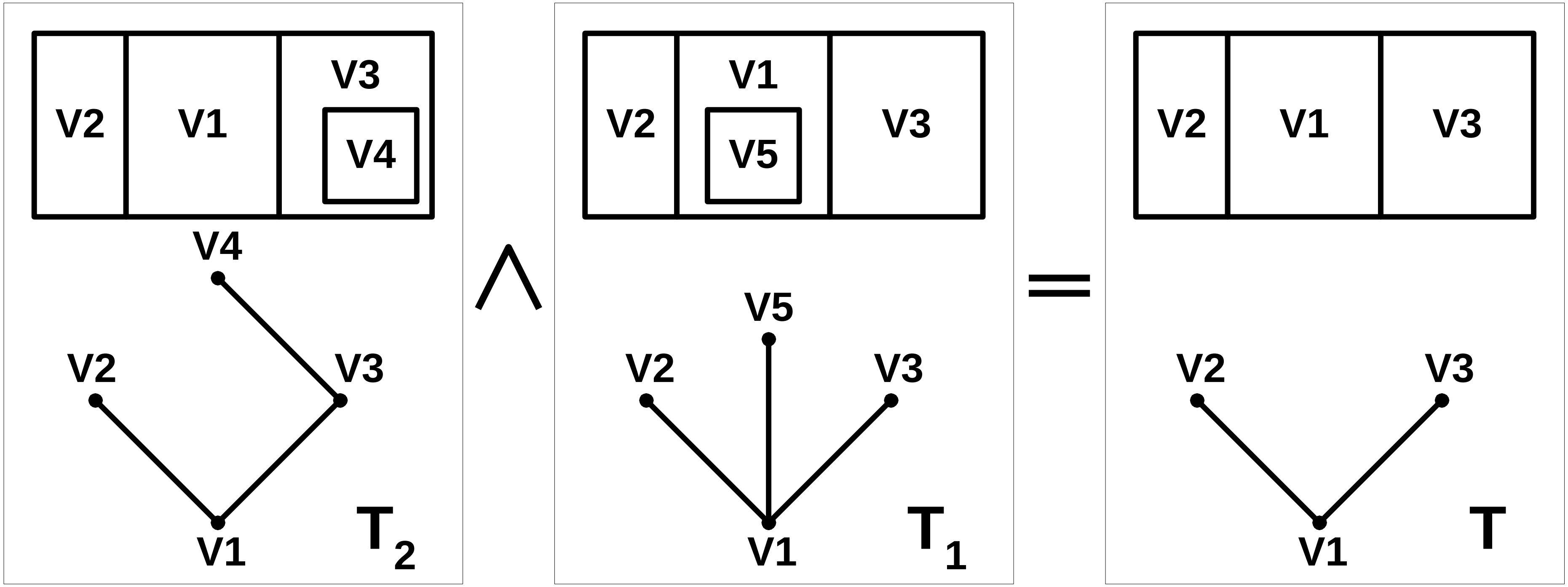
- For all $T_1=(t_1, M_1)$ and $T_2=(t_2, M_2)$

$$T_1 \leq T_2 \iff t_1 \subseteq t_2 \text{ and } M_1 \preceq_{t_2} M_2$$



The tree representation infimum

- The tree representation infimum is given by $T=(t, M)$
  - t is the infimum of the trees t_1 and t_2,
  - M is the infimum of the mapping functions M_1 and M_2.



Tree-domain morphological operators

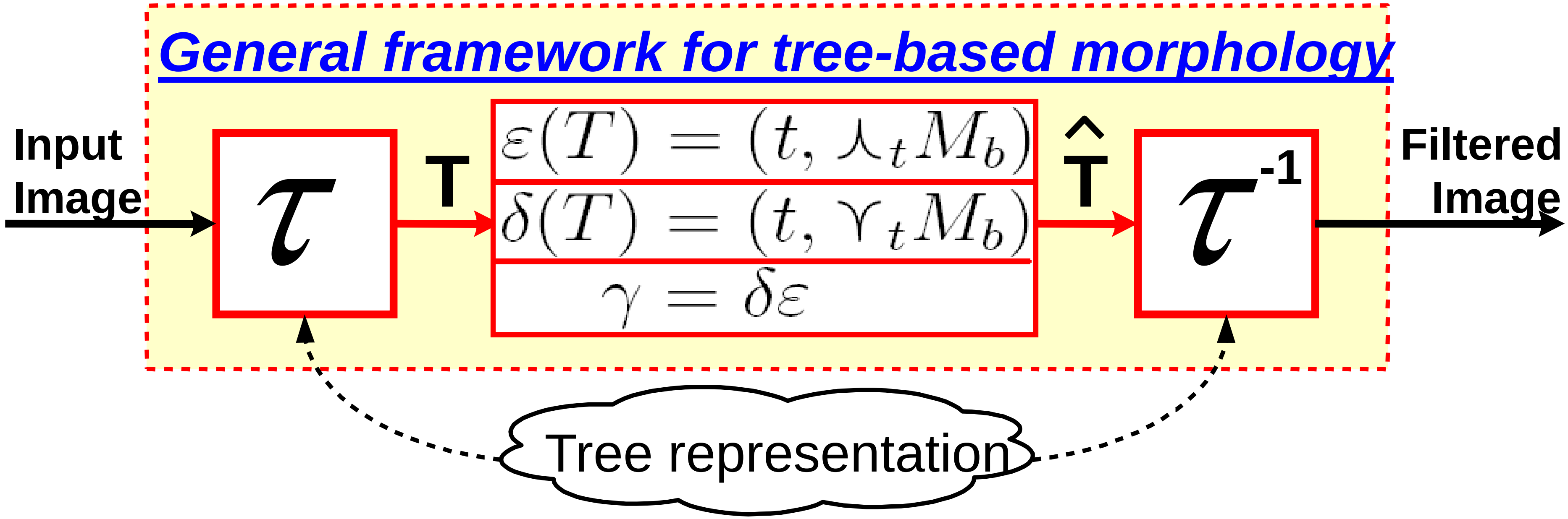
- For flat erosions & dilations:
  - Infima/suprema of translated maps
  - Same tree

$$\varepsilon(T) = (t, \curlywedge_t \{M_{-b} | b \in B\})$$

$$\delta(T) = (t, \curlyvee_t \{M_b | b \in B\})$$

$$\gamma = \delta\varepsilon$$

Tree-domain morphological operators



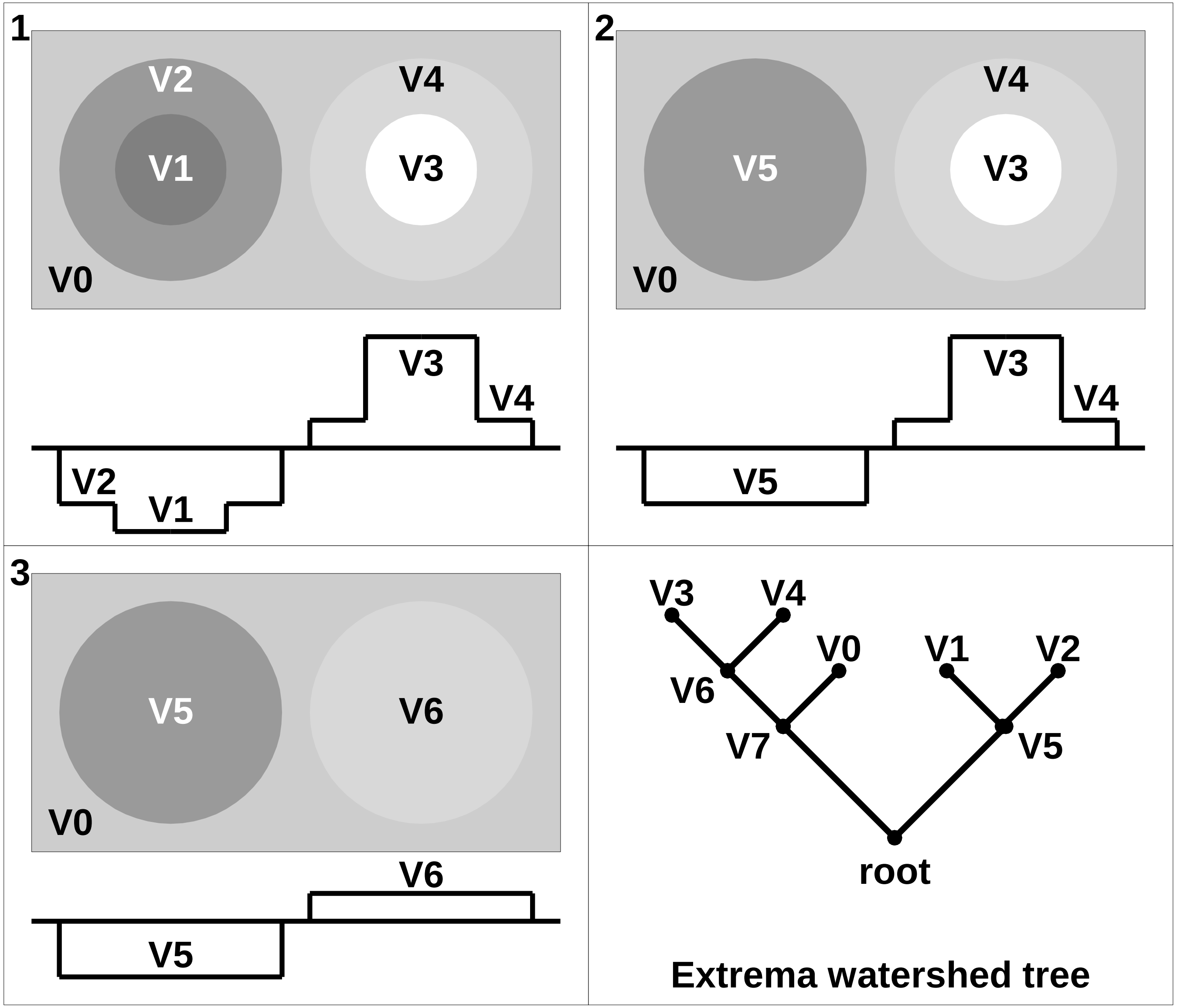
Semilattice of images

- What we would like to have: A semilattice of images, associated to each tree transform τ
  - The order of tree representation induces an order for images
  - However, the inf-semilattice of tree representations does not necessarily induce an inf-semilattice of images
  - For some trees (e.g., the tree of shapes) an image semilattice is obtained
  - For which trees we get an induced image semilattice? Issue under investigation

Extrema-Watershed Tree

- Example of new operators, obtained from the general framework
- Based on new self-dual tree-representation, called Extrema-Watershed Tree (EWT).
  - The tree is built by merging the flat zones.
  - Smallest extrema (dark or bright) regions are merged in every step.

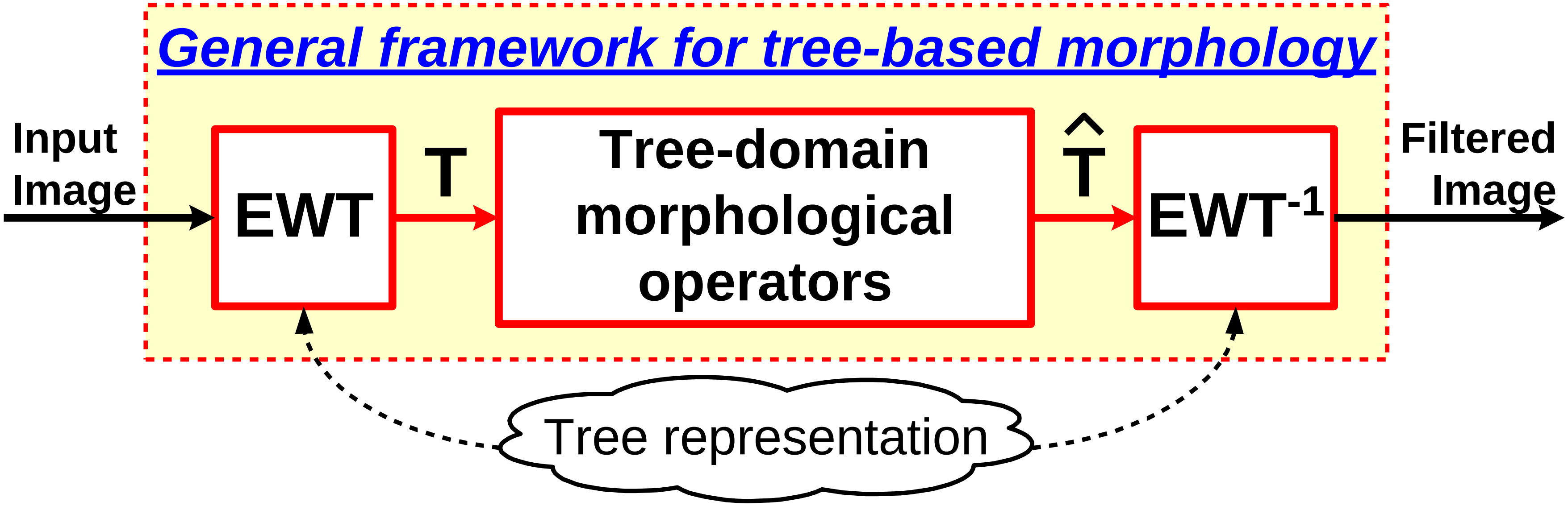
Building Extrema-Watershed Tree



Extrema watershed tree

EWT-based morphology

- EWT erosion and opening are obtained from the general framework.



Example: EWT Erosion

Original Image

EWT-based Erosion by SE 11x11

Properties of EWT

- Self-dual
- Implicit hierarchical decomposition
  - Tree is created in watershed-like process
  - Small area extrema are leaves
  - Bigger flat zones are close to root
  - Vertices connected in the tree usually have similar gray levels

Applications

- Self-dual morphological preprocessing.
  - Non-connected de-noising.
  - Opening by reconstruction:
    - Pre-processing for car license plate number recognition
    - Initial step for dust and scratch removal
- Potential for segmentation.

Filtering example

Noisy Source Image

Traditional opening-closing

EWT opening

Traditional closing-opening

Filtering example

Noisy Source Image

Opening by reconstruction EWT

Conclusions

- We presented a general framework for tree-based morphological image processing
  - Given a tree representation, corresponding morphological operators are obtained
  - Based on a complete inf-semilattices of tree representations
- Particular case
  - Self-dual morphology based on a new tree – EWT
  - Some applications are presented

Thank you!

Example 2

Original Image

Opened by reconstruction Image (EWT)

Pre-processing for car license plate number recognition

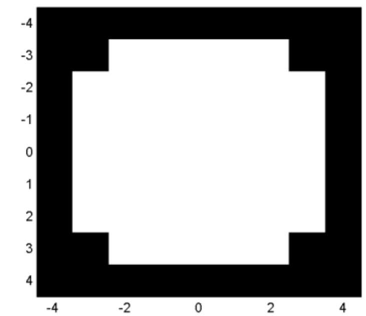
- The OCR algorithm includes coefficient of recognition quality, that enables to measure recognition improvements, when using different pre-processing algorithms.
- $I_{quasi-self-dual} = 256 - OR(256 - OR(I_{original}))$

Method used	**Recognition res.**	**Quality**
Without pre-processing	20-687-07	3.47
Averaging filter	70-587-02	3.65
Median filter	**70-587-07**	3.75
Regular opening by reconstruction	79-687-07	3.65
Regular quasi dual opening by reconstruction	**70-587-07**	3.79
EWT filter	**70-587-07**	**3.84**

License plate image corrupted with noise

License plate image in gray scale, as used by the OCR

SE



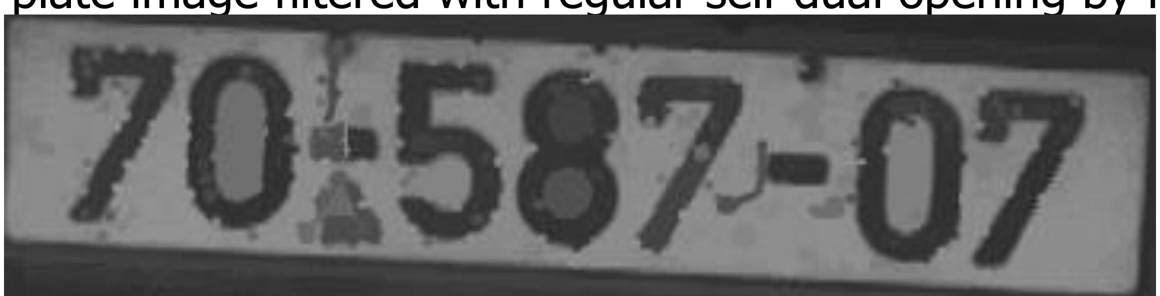
License plate image filtered by EWT-based opening by reconstruction

License plate image filtered with an averaging filter

License plate image filtered with a median filter

License plate image filtered with regular opening by reconstruction

License plate image filtered with regular self dual opening by reconstruction



Initial step for dust and scratch removal

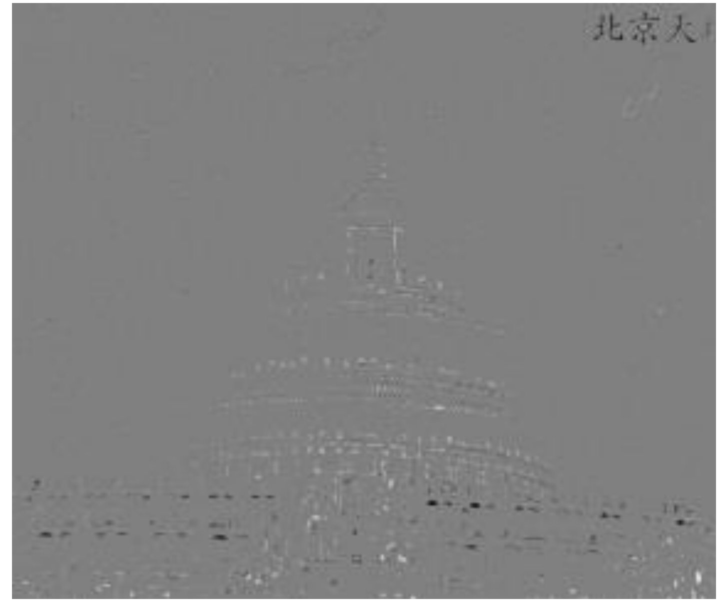
- Top-hat (TH) filter - includes all details that were filtered out by opening by reconstruction (OR): $TH = I_{original} - OR(I_{original})$
- As the energy level of the top hat image is lower, and as the dust and scratch removal is better, so the filter is declared to be more efficient.
  - Qualitative evaluation of the extent of dust and scratch removal
  - Energy level of the top-hat image: $Enrg = \|f\|_2 = \sqrt{\sum_{i,j \in f} f(i,j)^2}$

Method used	Cross SE	SE 3x3	SE 5x5
Averaging filter	1799	2643	3979
Median filter	**1233**	1971	3162
EWT filter	1376	**1737**	**2895**

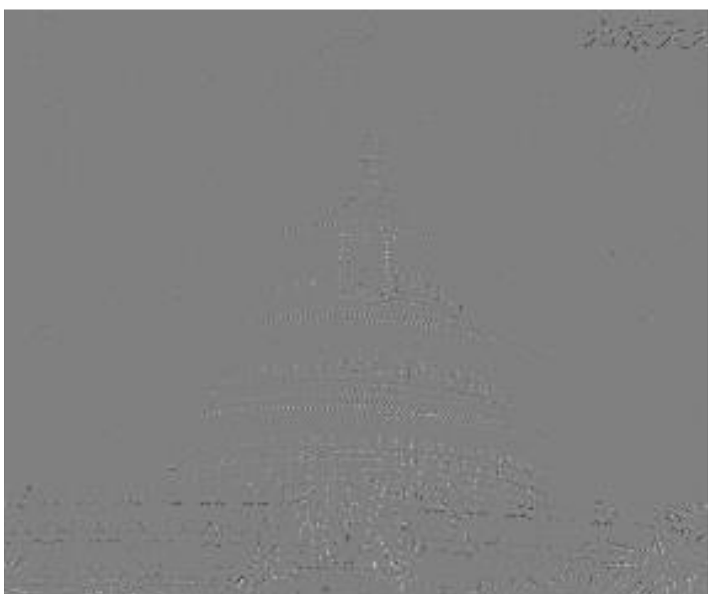
Cross SE

Original image

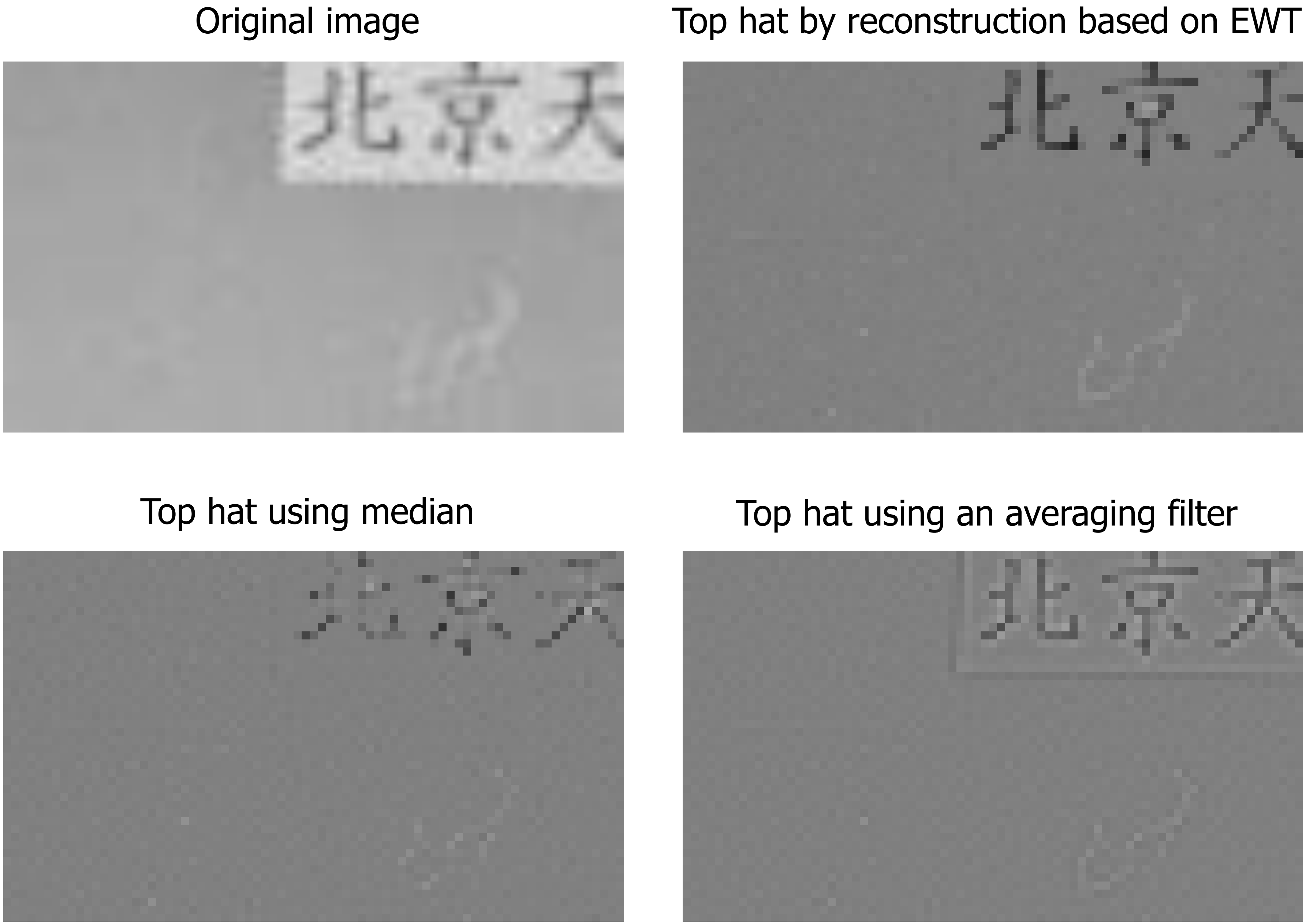
Top hat by reconstruction based on EWT



Top hat using median



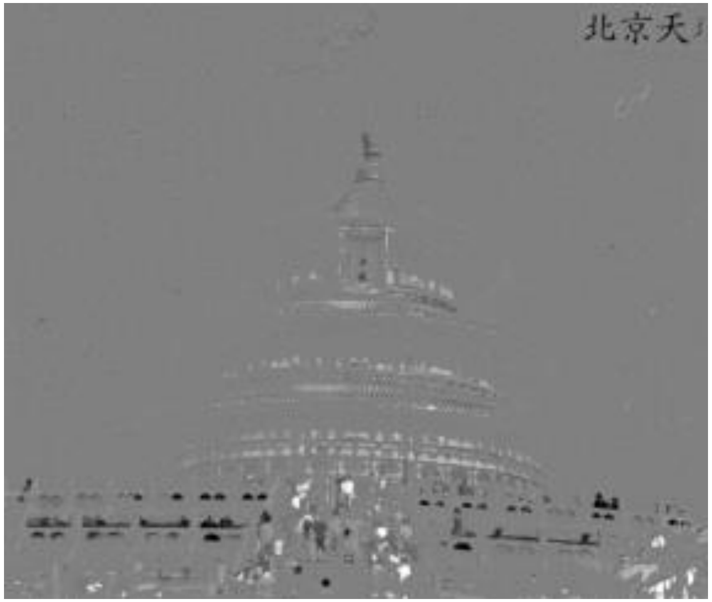
Top hat using an averaging filter


Original image
Top hat by reconstruction based on EWT
Top hat using median
Top hat using an averaging filter

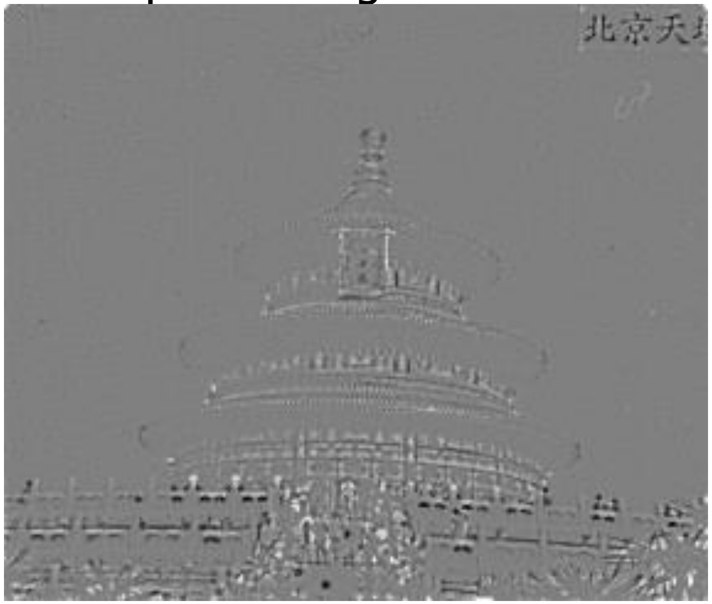
SE
5x5

Original image

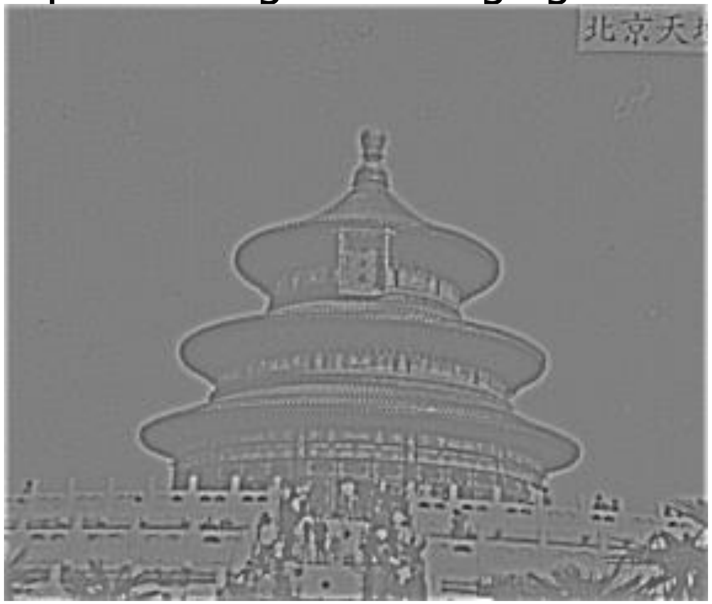
Top hat by reconstruction based on EWT

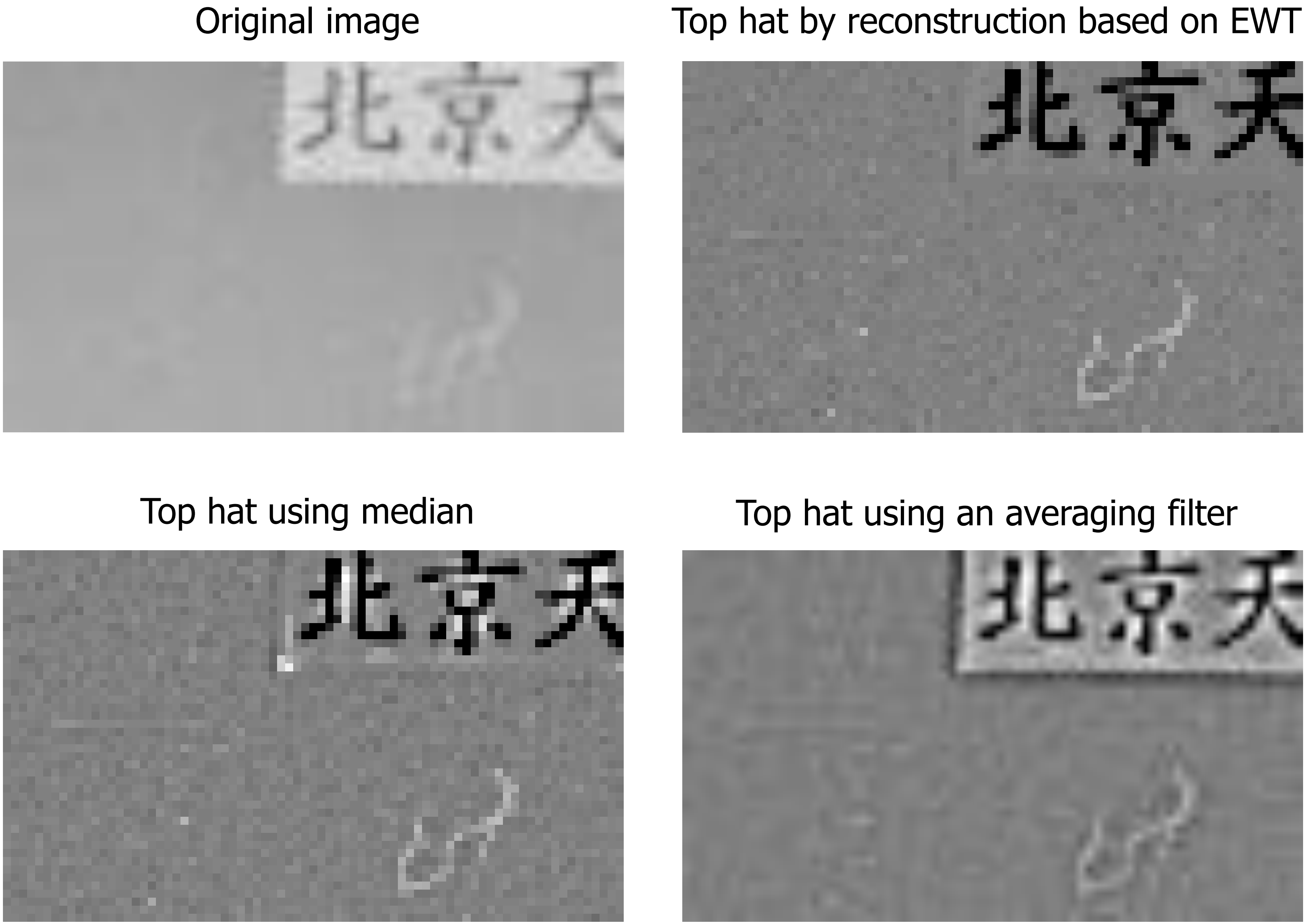


Top hat using median



Top hat using an averaging filter

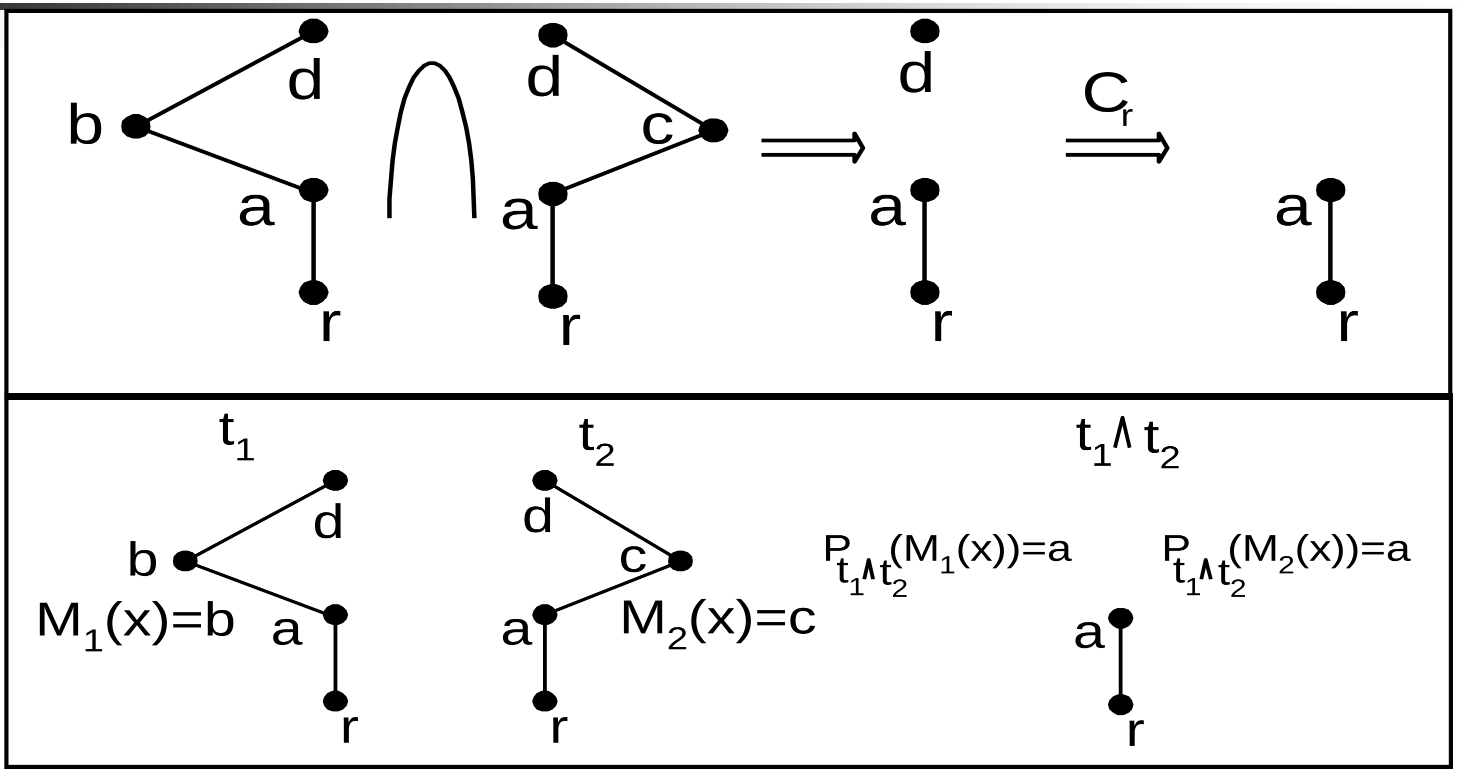



Original image
Top hat by reconstruction based on EWT
Top hat using median
Top hat using an averaging filter

Further research topics

- Finding necessary conditions for tree representation
  - To assure existence of images semilattice, induced by trees semilattice.
- More applications based on general framework.
  - Developing more useful tree representations
  - Further exploration of segmentation capability using trenches
- Real time implementation of the proposed algorithms.

The tree representation infimum



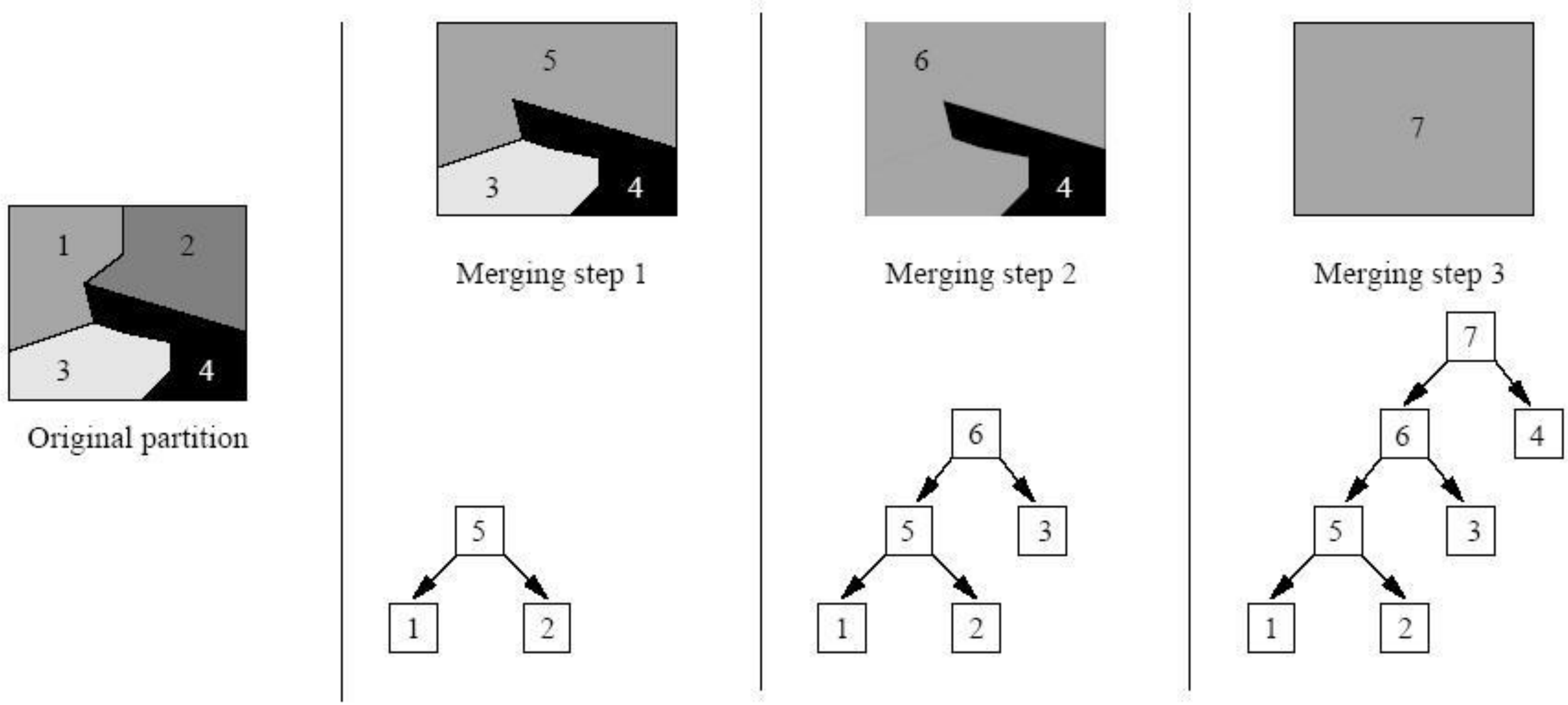
- Infimum of the trees is the intersection of the trees.
- For each point in E, the infimum mapping function is obtained by calculating the infimum vertex of the projections of the original mapping functions onto the infimum tree.

$$T_1 \wedge T_2 = (t_1 \wedge t_2, P_{t_1 \wedge t_2}(M_1) \curlywedge_{t_1 \wedge t_2} P_{t_1 \wedge t_2}(M_2))$$

Binary partition trees

(P. Salembier and L. Garrido, 2000)

- Are obtained from the partition of the flat zones.
- The leaves of the tree are flat zones of the image.
- The remaining nodes are obtained by merging.
- The root node is the entire image support.



Tree of Shapes

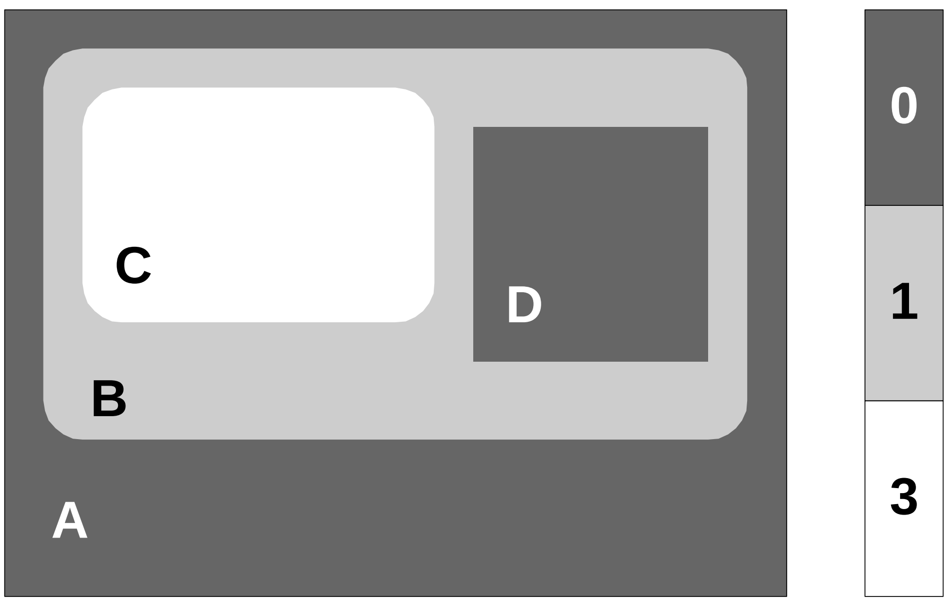
(P. Monasse and F. Guichard, 2000)

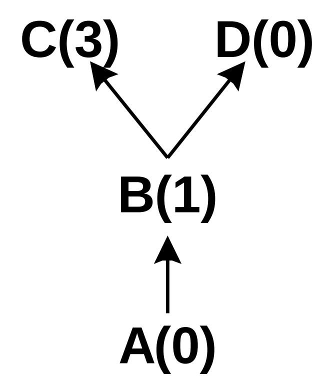
- Represents an image as hierarchy of shapes.
- Build according to the inclusion order.
- Each father vertex area includes also all sons area.
- Self-dual.

$$T_n(f) = \{x \in E \mid f(x) \geq n\}$$

$$FillHole(ConComp(T_n(f)))$$

$$FillHole(ConComp(T_n^C(f)))$$





Tree of Shapes