Hierarchical Fast Decoding of Fractal Image Representation Using Quadtree Partitioning

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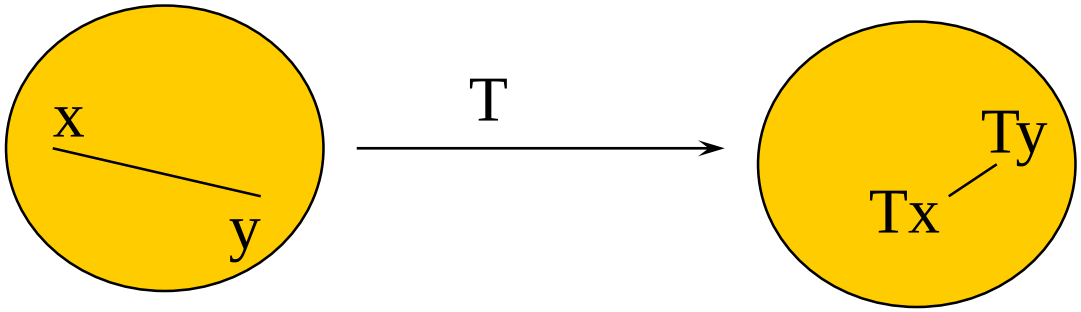
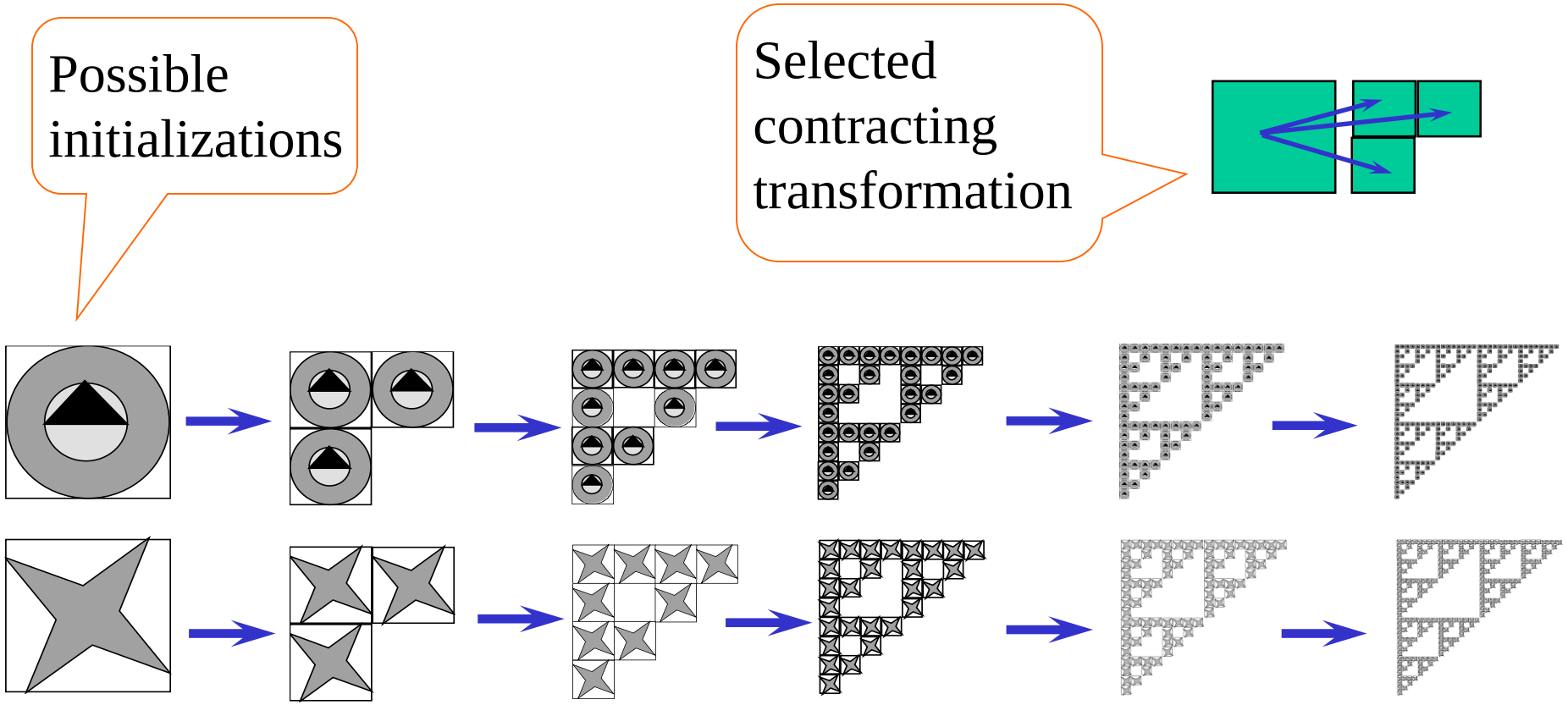
The Fractal Transform (background)

- A transformation T is called a 'contractor' if for all x and y : $\|Tx - Ty\| = s\|x - y\|$, $0 < s < 1$



- Such a transform has a unique fixed point $T(X_f) = X_f$ and $\lim_{n\to\infty} T^{(n)}(X_0) = X_f$ for every X_0.
- E.g., $T(x) = \frac{x}{2}$ is a contractor with $X_f = 0$.

Fractal Image Representation (Cont'd)



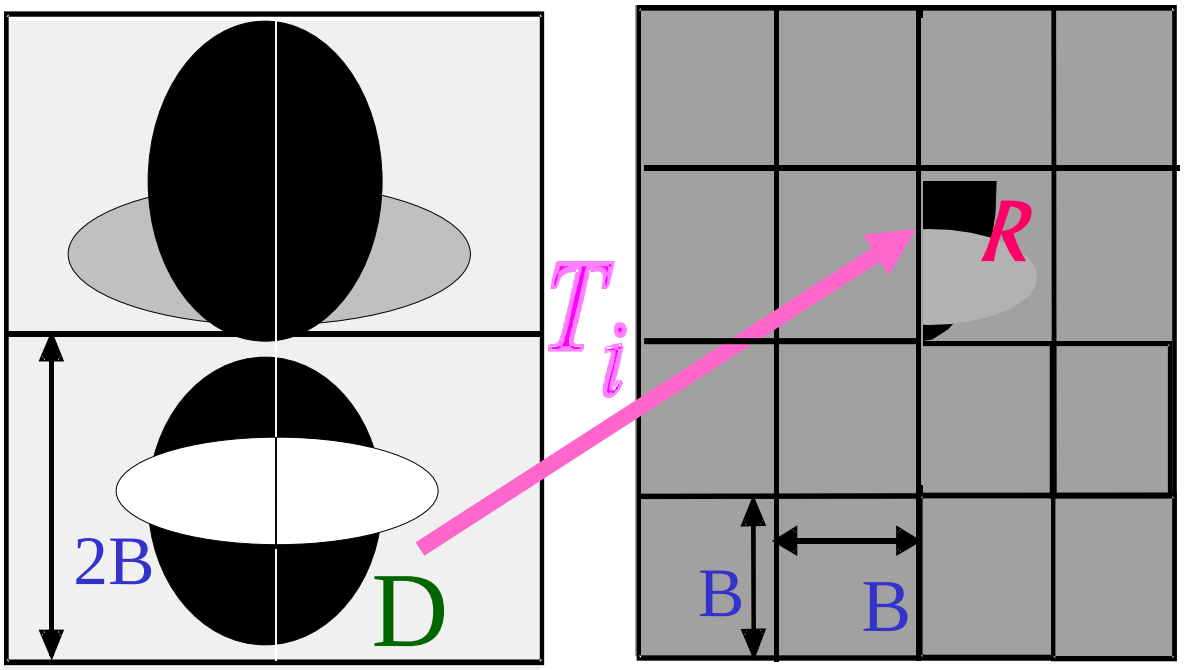
The fixed point is determined by the transformation and not by the initial images.

Fractal Image Representation (Cont'd)

- The inverse problem: what is the proper transformation T whose fixed point is a given image ?
- Is there a transformation T for each given image ?

Block-Based Fractal Coding

$$\hat{R}_i = \alpha_i \left(\varphi(D_{j_i}) - \overline{\varphi(D_{j_i})} \right) + \overline{R}_i$$



Domain

Range

φ is the contraction operator

α_i is the scale factor

$\overline{R_i}$ is a block of constant value equal to the mean of the range block R_i

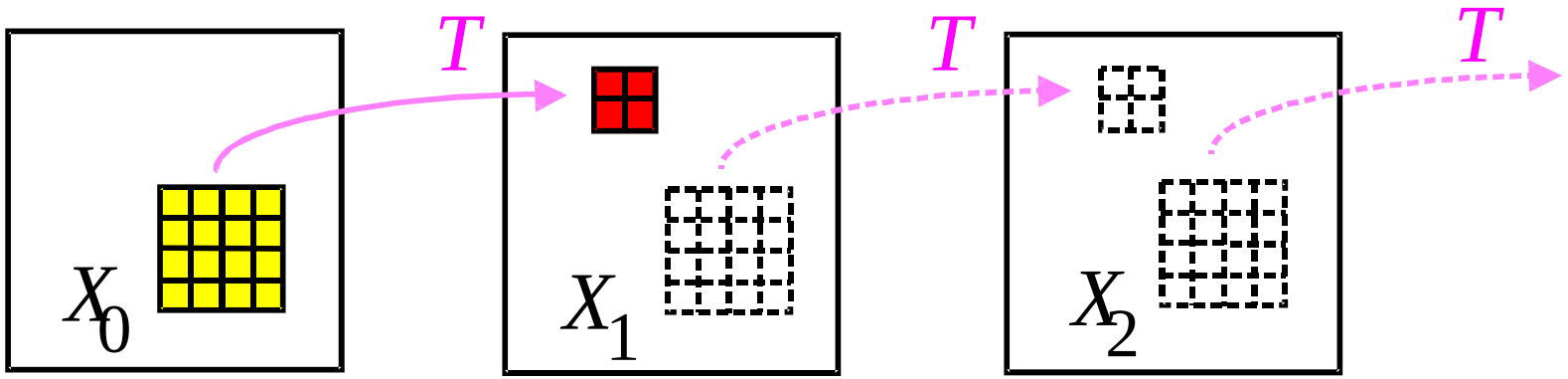
D_{j_i} is the domain block best fitting R_i

Fractal Image Coding [Jacquin 1992]

- The quantized linear estimation parameters are the parameters that form the code.
- Solving the inverse problem - according to the collage theorem

$$\left\| X - X_f \right\| \le \frac{1}{1-s} \left\| X - T(X) \right\|$$

- Decoding: Iteratively applying the transformation T to any initial image.



An Example

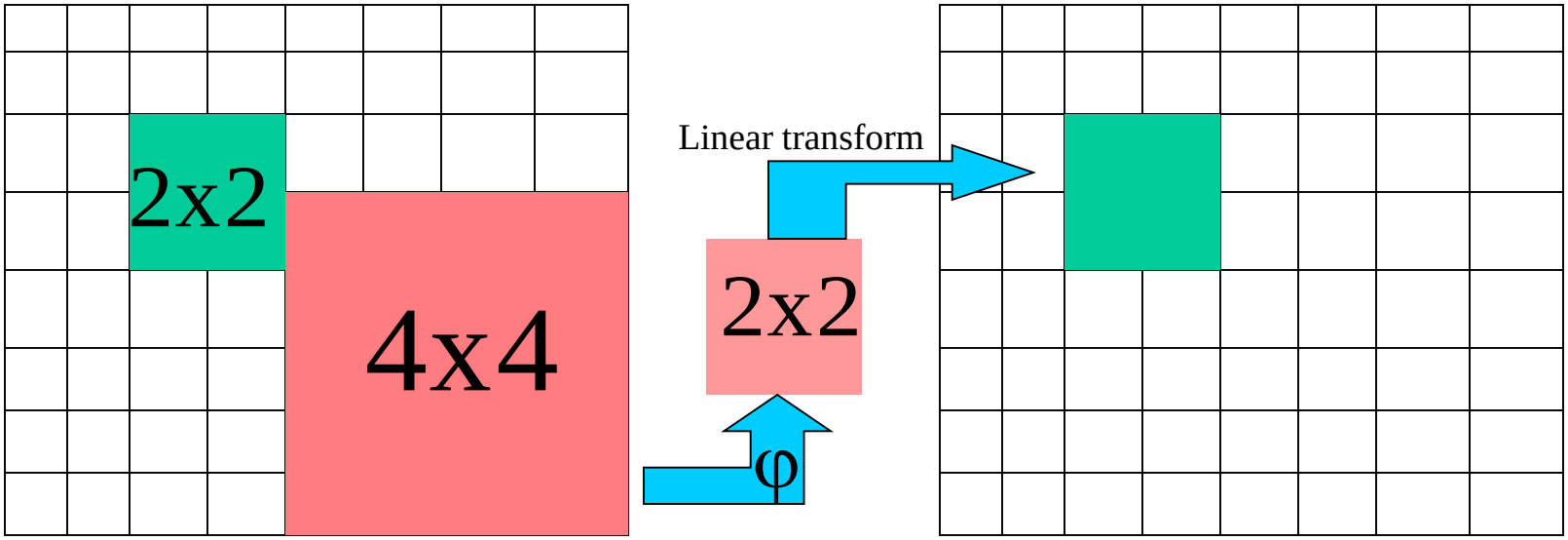
Hierarchical Decoding

- Jacquin's decoder can only estimate the fixed point, because it uses a finite number of iterations.
- Finite precision may prevent convergence.

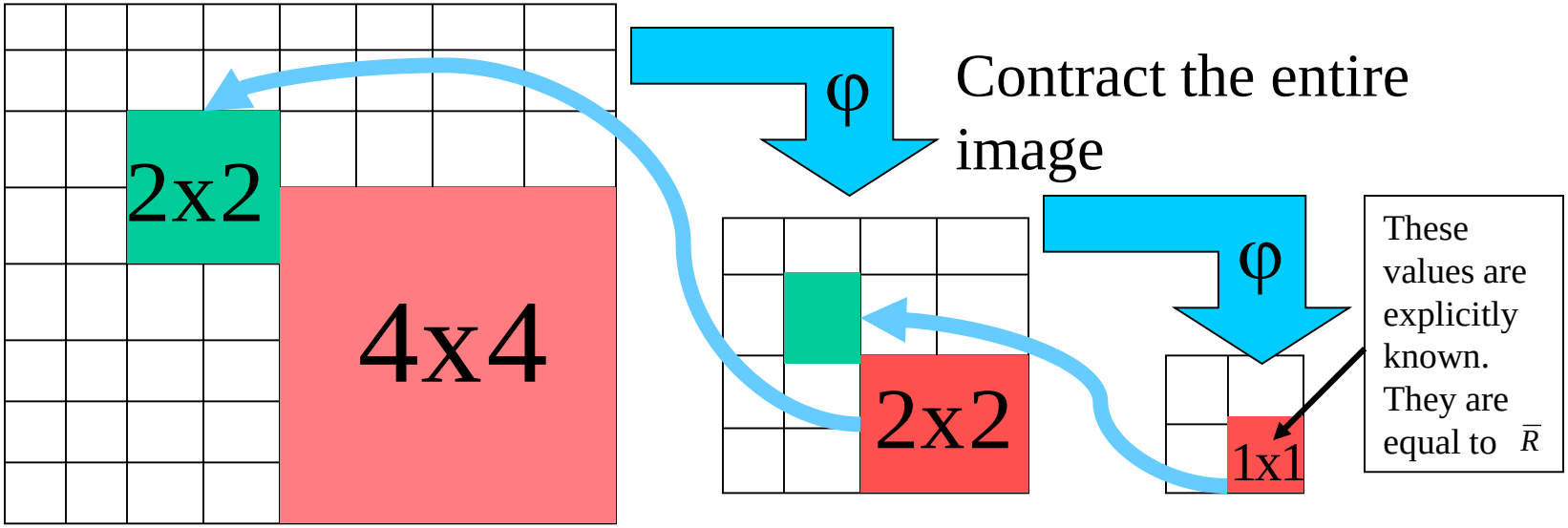
- Hierarchical decoding was proposed by Baharav [Baharav 1993] & ϕien [Lepsϕy, ϕien, Ramstad 1993].
- It uses a finite number of steps , $\log_2(B)$, to reach the (true) fixed point.

The Hierarchical Representation

Instead of iterating:

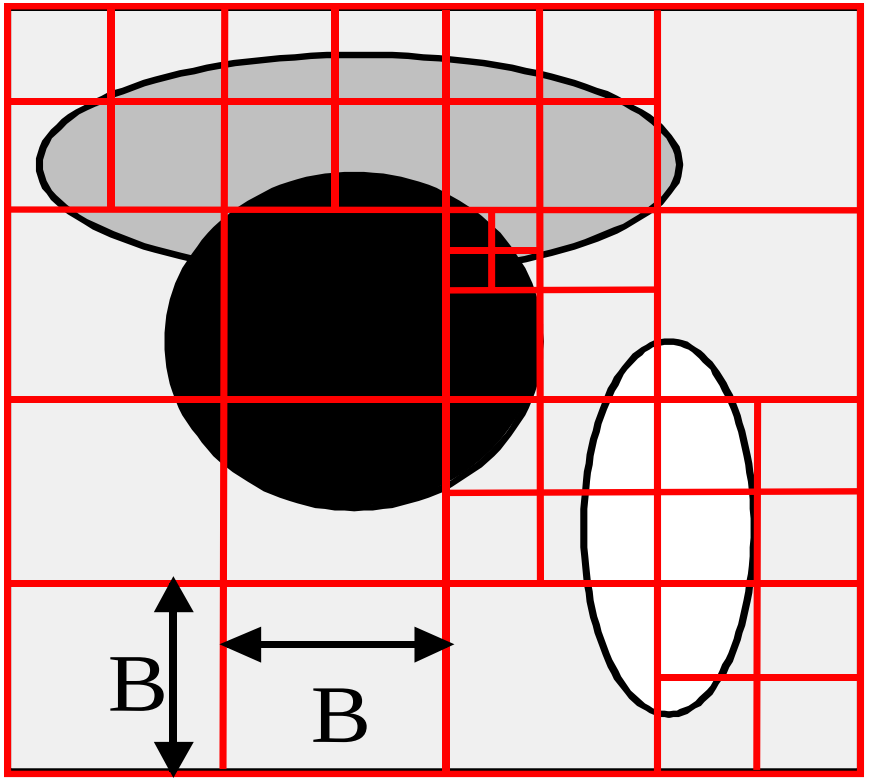


Use:



Encoding with Quadtree Partitioning

- The size of the range blocks is adaptively chosen to maintain high quality image at a low bit rate.
- The structure of the tree is transmitted to the decoder.



Problem Statement

- How can hierarchical decoding be applied to quadtree partitioned images?
- To stress the point: which B to use? and how? when different range blocks may have different sizes.
- Sub-problem: how is the transformation defined at low resolutions? (some range blocks are not available any more).

Hierarchical Fast Decoding with Quadtree Partitioning

- Hierarchical representation still exists.
- A range block at a low resolution may be either
  - a contracted version of a range block at the highest resolution
  - a $1x1$ block with a value that is a weighted average of the mean values of a union of range blocks at the highest resolution.

The Generalized Theorem

- Let $t_i = \{\alpha_i, j_i, r_i\}$ be local transformations and let $T_1 = \bigcup_i t_i$ be the fractal transform (including the quadtree structure).
- Let X_1 be the fixed point of T_1.
- Let $X_{\frac{1}{2^m}}$ be the m-th contraction of X_1 .
- Assume:
  - All range blocks are $2^k \times 2^k$ for different $k \geq 0$
  - For every B such that a $B \times B$ range block exists, there is a pool of domain blocks such that the displacement between blocks is B.

The Generalized Theorem (Cont'd)

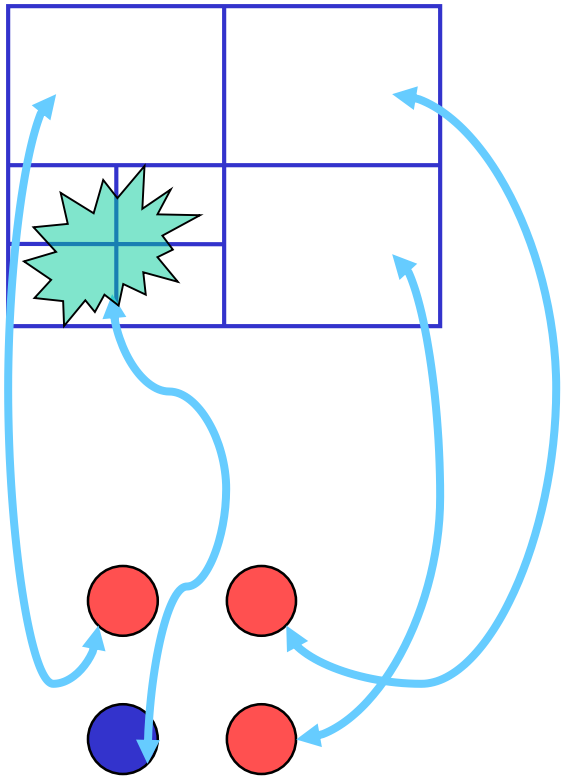
- Then, there is a way of obtaining transformations $T_{\frac{1}{2^m}}$ directly from T_1 such that

$$X_f\left\{T_{\frac{1}{2^m}}\right\} = X_{\frac{1}{2^m}}$$

- In the case of a fixed grid partition

$$T_1 = T_{\frac{1}{2^m}} \quad \text{for} \quad 0 \le m \le \log_2(B)$$

Generalized Hierarchical Fast Decoding

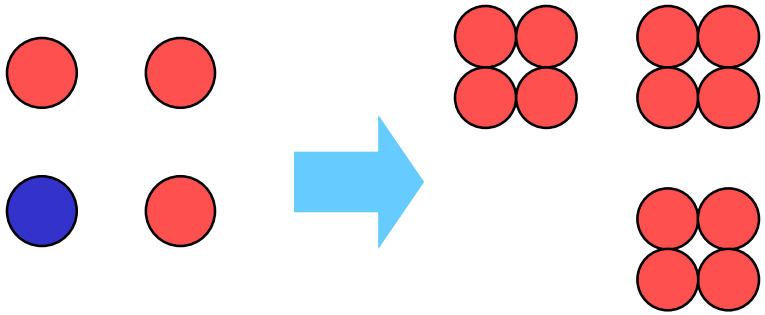


An example of hierarchical fast decoding:

According to the given quadtree partition

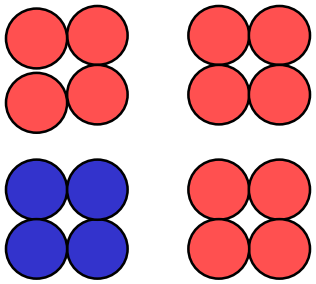
1. Initialization: Every pixel corresponds to a block of the maximal size. Its value is the block's mean value, given explicitly or implicitly in the code.

Generalized Hierarchical Fast Decoding (Cont'd)



source

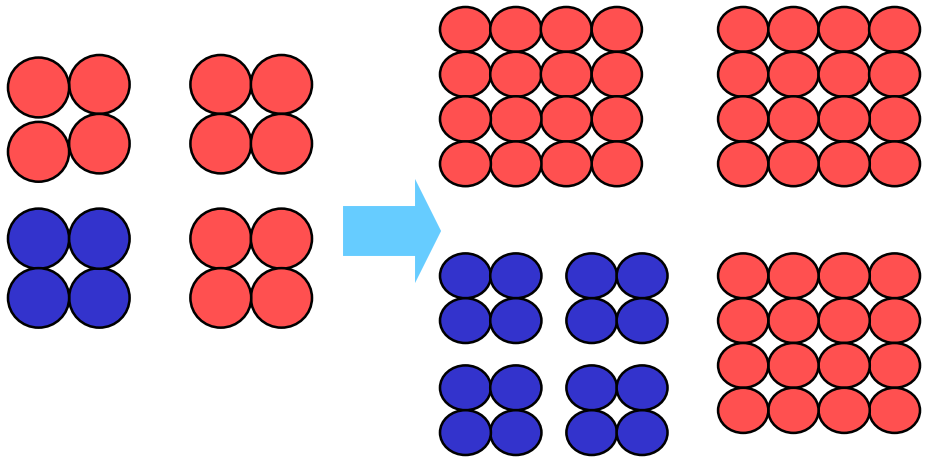
destination



<u>At every step:</u>

2. For every pixel at the source image that corresponds to a range block, perform "Zoom-In" using the fractal transformation.
3. Otherwise, implant the proper mean values as at stage 1.

Generalized Hierarchical Fast Decoding (Cont'd)



source destination

4. Repeat steps 2-3 until the destination image is of the size of the original image.

   A total of $\log_2(B)$ iterations is performed, where B is the size of the maximal range block.

   Each range block is obtained by log(its size) iterations.

Demonstration

Decoding process

Reconstructed image

Quadtree partitioning

Discussion and Conclusions

- The generalized fast decoding algorithm was proved to reach the fixed point of the transformation T in a finite number of steps, even if quadtree partitioning is used.
- Proof is obtained by induction.
- The proper fractal transform at a low resolution may be derived from the original fractal transform.
- Limitations on scale parameter may be abandoned.
- Hierarchical decoding cannot be applied to any image partitioning method.