On Joint Distribution Modeling in Distributed Video Coding Systems

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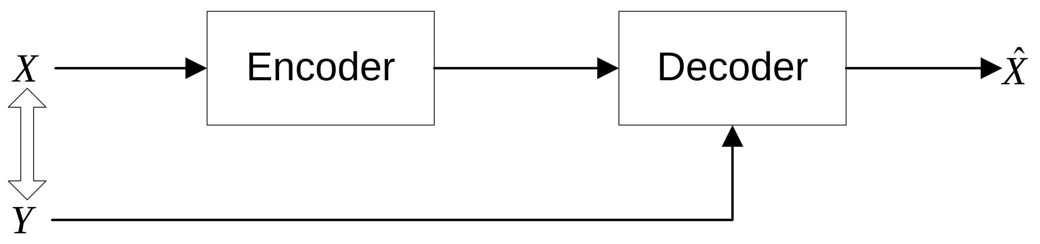
1. Introduction

1.1 Motivation



- Joint distribution of Source X and Side Information Y is required for Slepian-Wolf decoding and de-quantization
- In DVC the joint distribution of X and Y is not known
- Accurate modeling of the joint distribution reduces coding rate and distortion

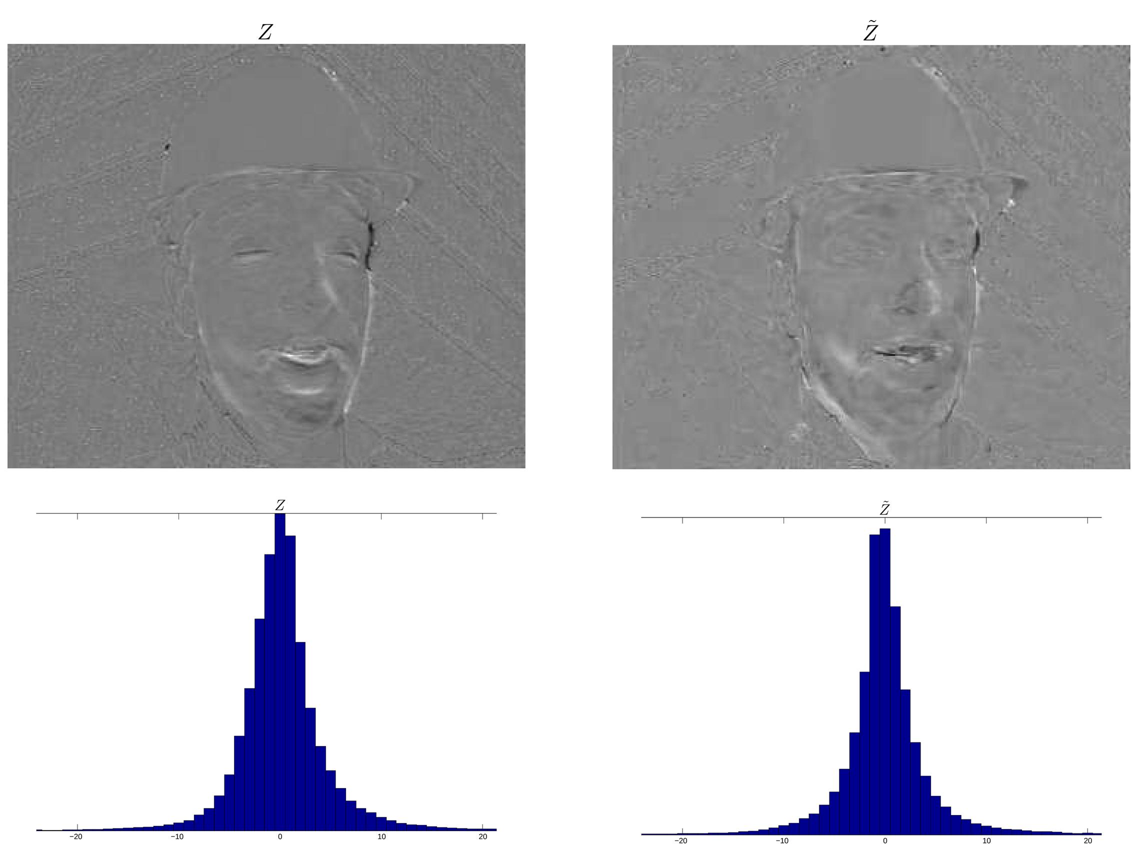
1.2 Background

- Off-line modeling
  - Widely adopted model assumes that: $Z = Y - X$, $Z \sim Laplace(\mu = 0, \sigma)$
  - Typical, distribution parameters are learned off-line based on a set of test sequences
  - This approach fails to capture temporal variations
- On-line modeling
  - X available only at the encoder, Y available only at the decoder
  - The noise image Z is approximated using decoded Key or WZ frames

$$\tilde{Z}(x,y) = \frac{1}{2}(X_b(x+dx_b, y+dy_b) - X_f(x+dx_f, y+dy_f))$$

  - Approximation accuracy depends on ME algorithm, GOP size and reference frames quality

1.3 Example: True and Approximate Noise Images



2. Stationary Modeling

2.1 DCT Virtual Channel - Assumptions

- Noise is additive ($Z = Y - X$), white and independent of Y
- Noise distribution in each DCT band is characterized by a different set of parameters

2.2 Generalized Gamma Distribution

$$f_Z(z; a, v, p) = \frac{pa^{-pv}}{2\Gamma(v)} |z|^{pv-1} e^{-(|z|/a)^p},$$

$$z \in \mathbb{R}, \quad a, v, p > 0$$

2.3 Sub-distributions

	Parameters	# Parameters
Gen. Gaussian	vp=1	2
Gamma	p=1, v=0.5	1
Laplace	p=1, v=1	1
Gaussian	p=2, v=0.5	1

2.4 Model Comparison

- The fit of various distributions was evaluated using AIC and MDL metrics
- Both metrics penalize models with large number of parameters

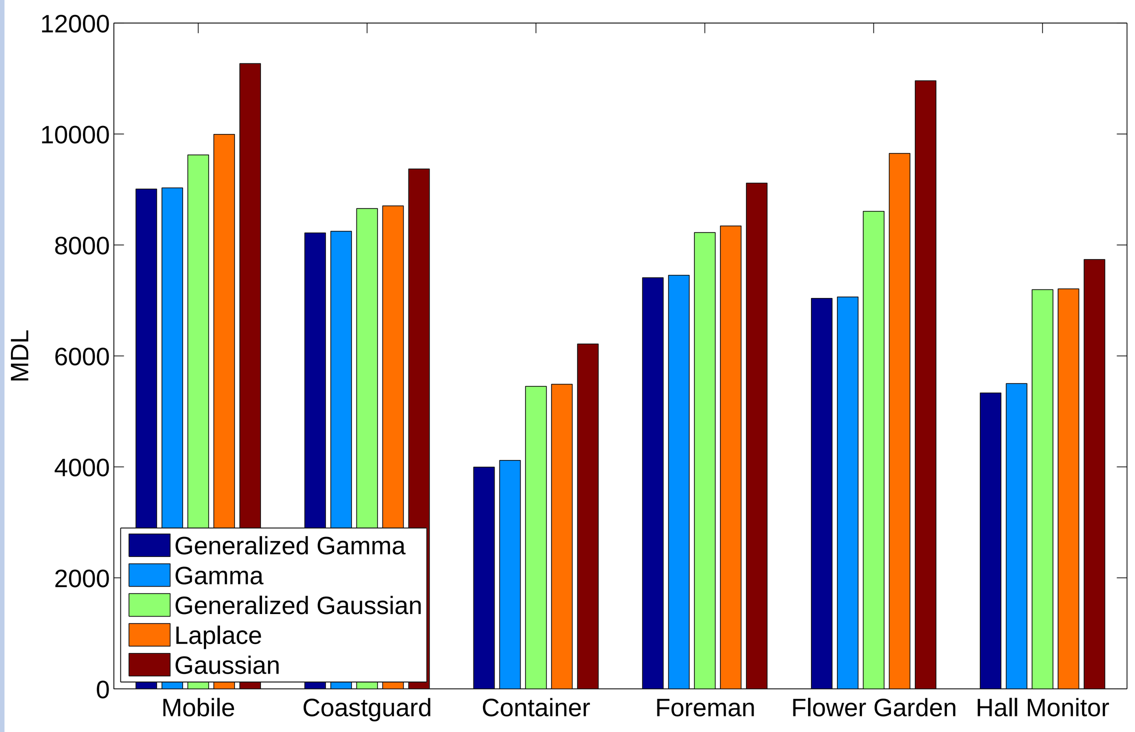
$$AIC(\hat{\theta}_{ML}) = -2\log\left[f\left(\underline{x}; \hat{\theta}_{ML}\right)\right] + 2k$$

$$MDL(\hat{\theta}_{ML}) = -\log\left[f\left(\underline{x}; \hat{\theta}_{ML}\right)\right] + \frac{k}{2}\log(n)$$

$\hat{\theta}_{ML}$ – Maximum Likelihood estimate of model parameters

k – the number of parameters

2.6 MDL Fit for QCIF Sequences



2.7 Selecting a New Model

- MDL and AIC metrics for Gamma are very close to GΓD
- The double sided Gamma distribution has a single parameter:

$$f(z) = \frac{1}{2\sqrt{\pi a|z|}} e^{\frac{-|z|}{a}}, \qquad a > 0$$

- Closed form ML estimator: $\hat{a}_{ML} = \frac{2}{n}\sum_{i=1}^{n}|x_i|$

3. Spatially Adaptive Modeling

3.1 Model Localization

- Stationary models fail to capture the spatially varying joint distribution
- In order to capture the spatial variation, the modeling and estimation should be performed in the pixel domain
- Spatially adjacent virtual channel pixels are correlated

3.2 MVL Distribution

- No evidence could be found that the family of multivariate double sided Gamma distribution is closed with respect to linear transformation
- Use MultiVariate Laplace (MVL) distributions to model $n_b \times n_b$ virtual channel blocks:

$$f(z) = 2(2\pi)^{-\frac{d}{2}}|\Sigma|^{-\frac{1}{2}}\left(\frac{z'\Sigma^{-1}z}{2}\right)^{\frac{v}{2}} K_v\left(\sqrt{2z'\Sigma^{-1}z}\right),$$

$z \in \mathbb{R}^d$, $d = n_b^2$, $v = (2-d)/2$ and $K_v(\cdot)$ - modified Bessel function

- MVL Characteristic Function

$$\Phi(t) = \frac{1}{1 + \frac{1}{2}t'\Sigma t}, \qquad t \in \mathbb{R}^d$$

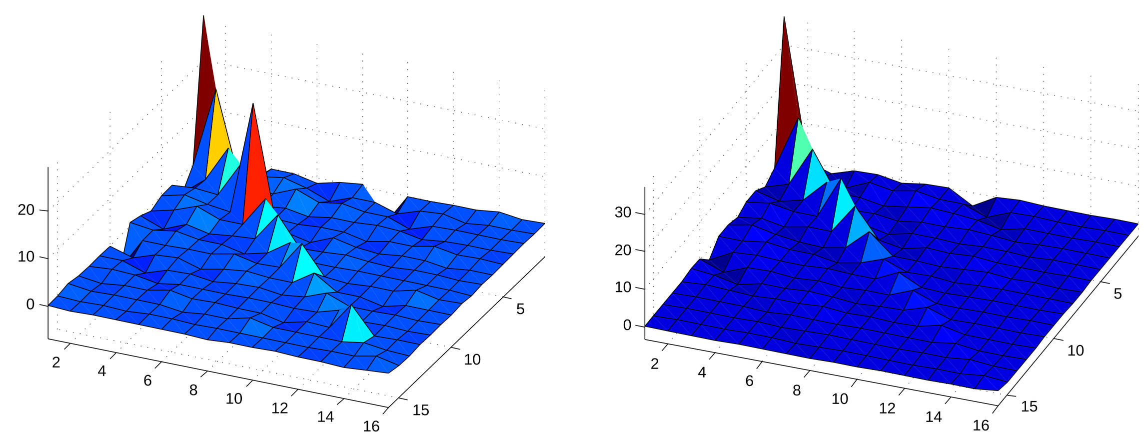
3.3 MVL Properties

- If $Z = z_1, \ldots, z_d$ is an MVL distribution then 1D marginal distributions of Z are Laplace, $z_i \sim Laplace(\mu = 0, \sigma_{ii})$
- If $Z \sim MVL(\Sigma)$ then $W = AZ \sim MVL(A'\Sigma A)$:

$$\Phi_W(t) = E[e^{iWt}] = E[e^{i(AZ)'t}]$$
$$= E[e^{iZ'A't}] = \Phi_Z(A't)$$

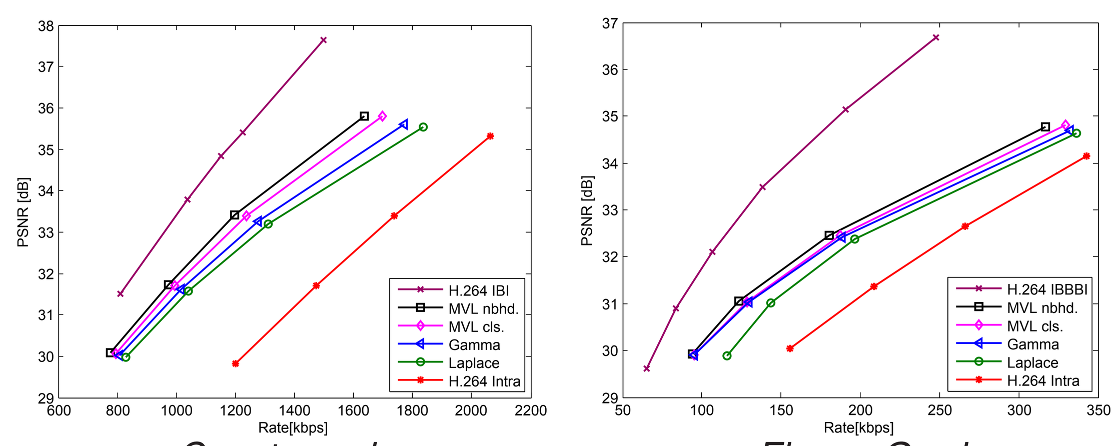
3.4 MVL Distribution Parameters Estimation

- Pixel domain block-wise autocovariance matrix, Σ, estimation using samples from $n_w \times n_w$ window
- Transform Σ for each block to DCT domain
- Typically, cross-band elements of Σ are relatively small. Thus, 1D marginals can be used without significant loss
- Results in univariate Laplace distribution with a different, spatially dependent, parameter value for each DCT coefficient



DCT domain autocovariance of two 4×4 blocks

4. Simulation Results



Coastguard (QCIF, GOP 4, 15fps) — *Flower Garden* (CIF, GOP 2, 15fps)

- For stationary modeling rate reduction of 2.5-4% for Flower Garden and of 2-18% for Coastguard were obtained
- For spatially adaptive modeling rate reduction of 6-10% can be observed for Flower Garden and 6-19% for Coastguard

5. Summary

- This work addresses the problem of model selection for joint distribution modeling in DVC systems
- A modeling technique which overcomes the non-valid assumption of spatial stationarity is presented
- The proposed double-Gamma and MVL models outperform the commonly used stationary Laplace model

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