Context-Based Multiple Description Wavelet Image Coding

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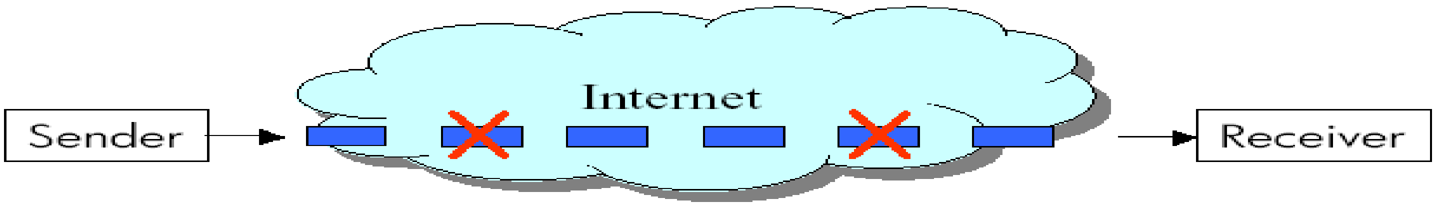
Outline

- Fundamentals of Multiple Description (MD) coding
- Framework: MD coding via polyphase transform and selective quantization
- Proposed system:
  Context-based MD wavelet image coder
  - Motivation
  - Detailed description
- Experimental results
- Summary

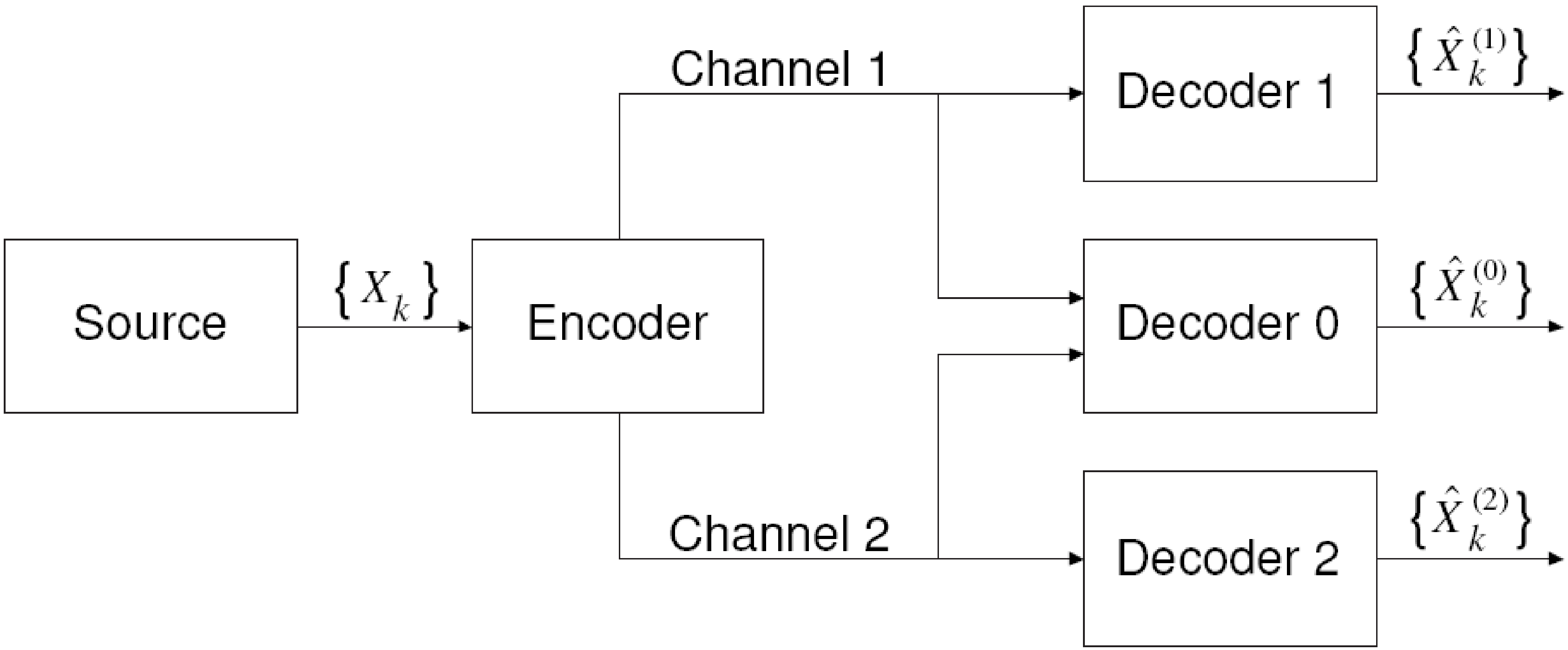
Fundamentals of MD Coding: Introduction



- Purpose: Provide error resilience to information transmitted on lossy networks (e.g., the Internet)
- Possible solution - MD coding:
  - Represent the information source with several descriptions
  - The source can be approximated from any (non-empty) subset of the descriptions

⇒ Makes all received descriptions useful

Fundamentals of MD Coding: Scenario for MD Coding (Two Descriptions)

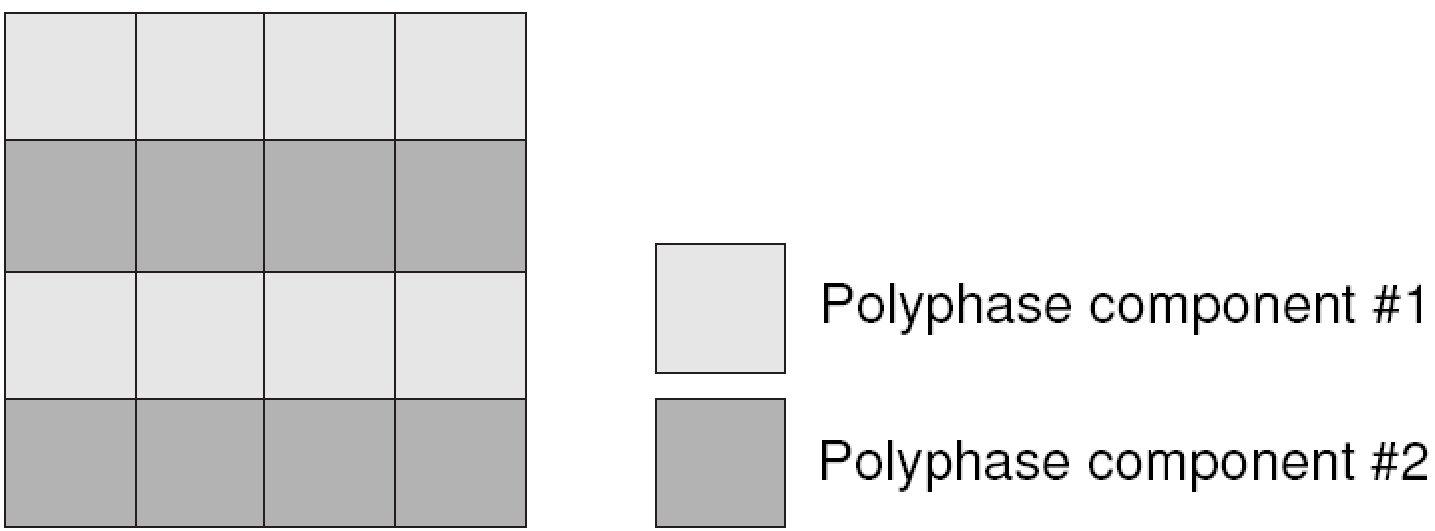


Framework: MD Coding via Polyphase Transform and Selective Quantization

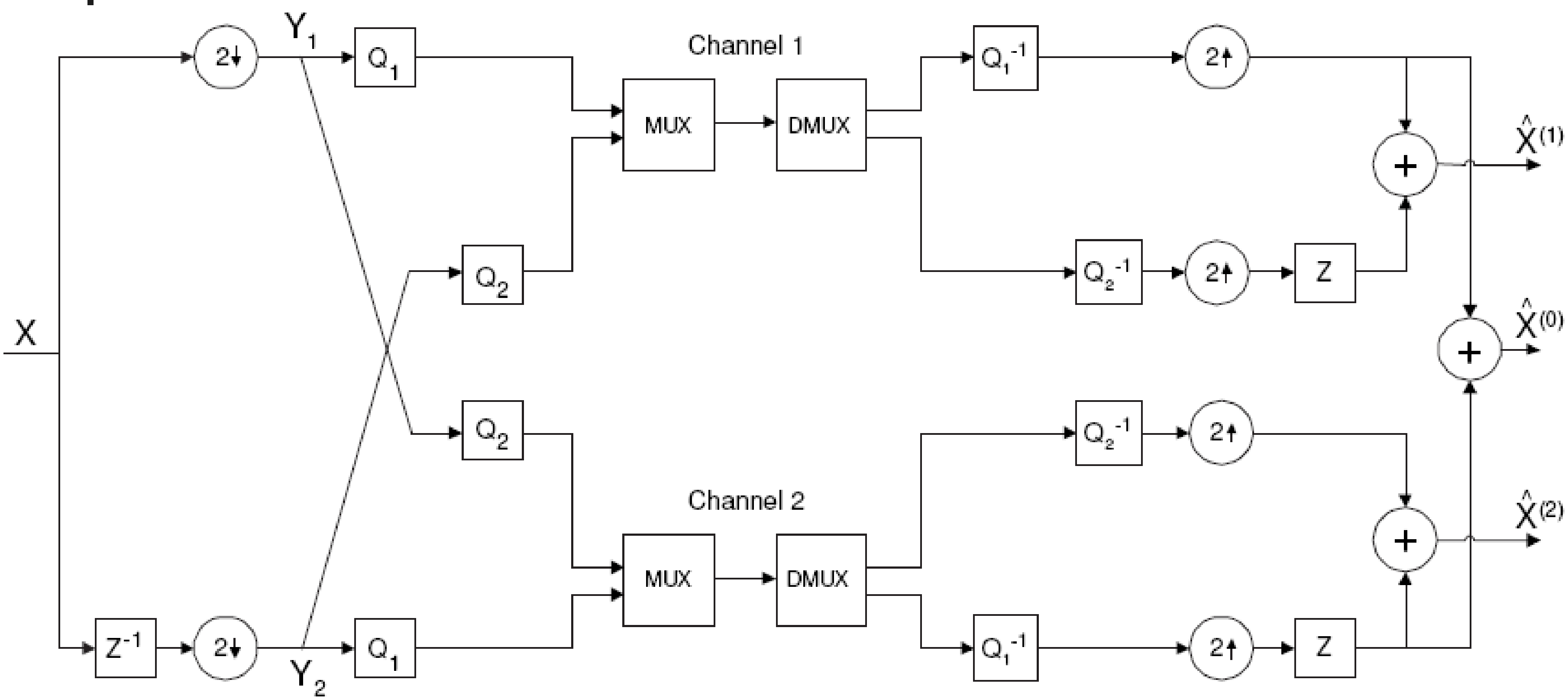
- Proposed by [Jiang and Ortega, 1999]
- Explicitly separates description generation and redundancy addition
  - ⇒ Reduced complexity of design and implementation

Polyphase Transform-Based MD Coding: The Polyphase Transform

- Polyphase transform:
  Decomposition to polyphase-like components
- Example – plain polyphase transform:



Polyphase Transform-Based MD Coding: System Outline

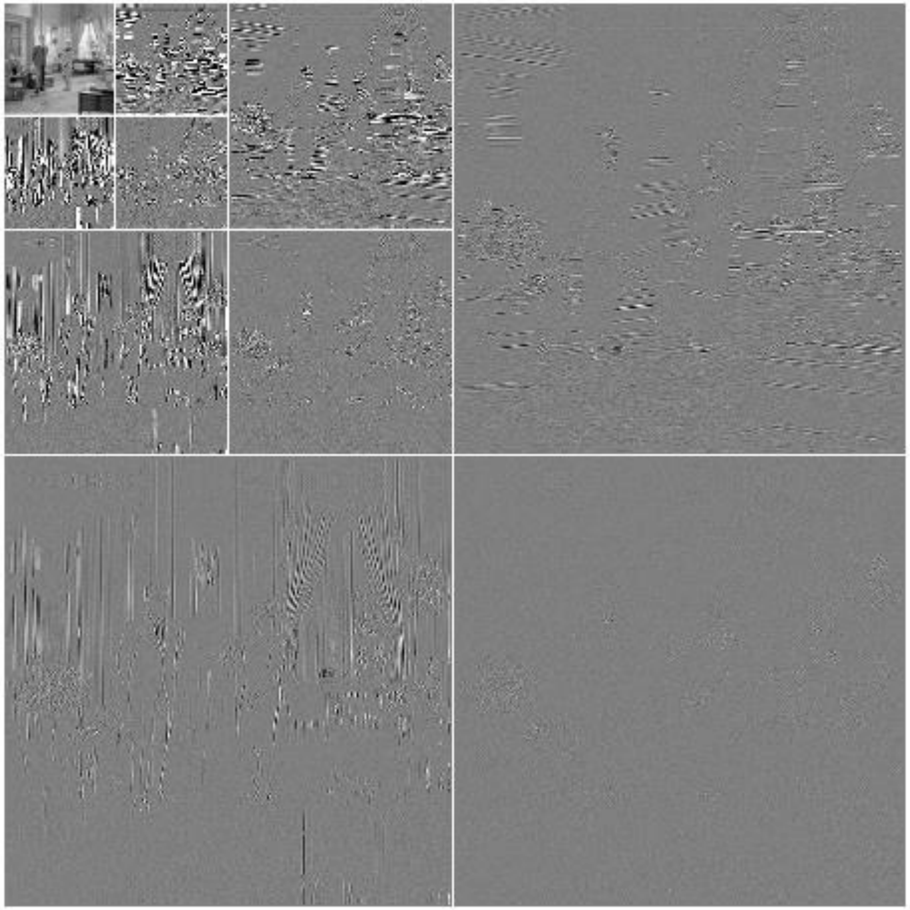


- For correlated input data (e.g., an image), a preliminary decorrelating transform (e.g., wavelet transform) is required

Wavelet Background: Statistical Characterization of Wavelet Coeffs

- Example:

Original



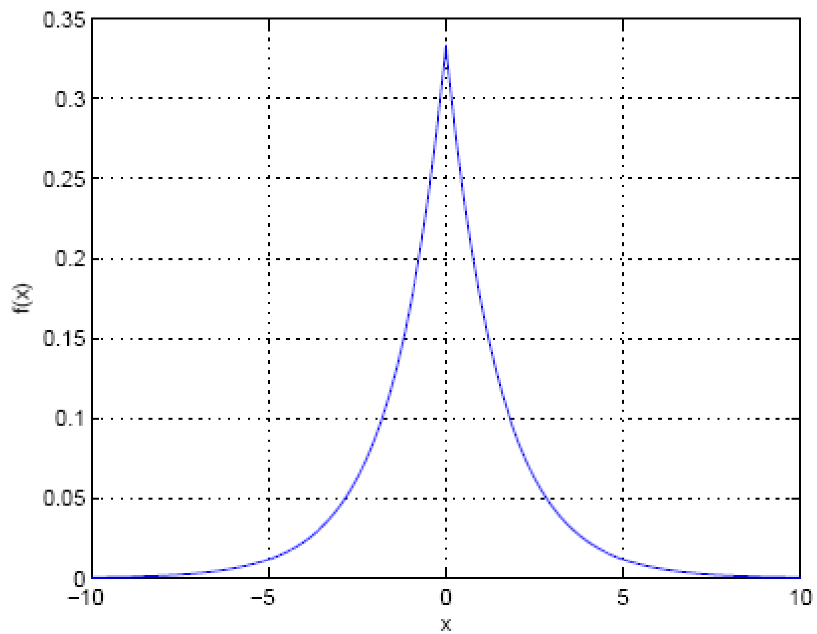
Wavelet transform

Wavelet Background: Statistical Characterization of Wavelet Coeffs (cont.)

First Order Statistics:

- First order statistics of detail wavelet coeffs can be reasonably modeled using Laplacian distribution [Buccigrossi and Simoncelli, 1999]

$$f_\lambda(x) = \frac{\lambda}{2} e^{-\lambda |x|}$$



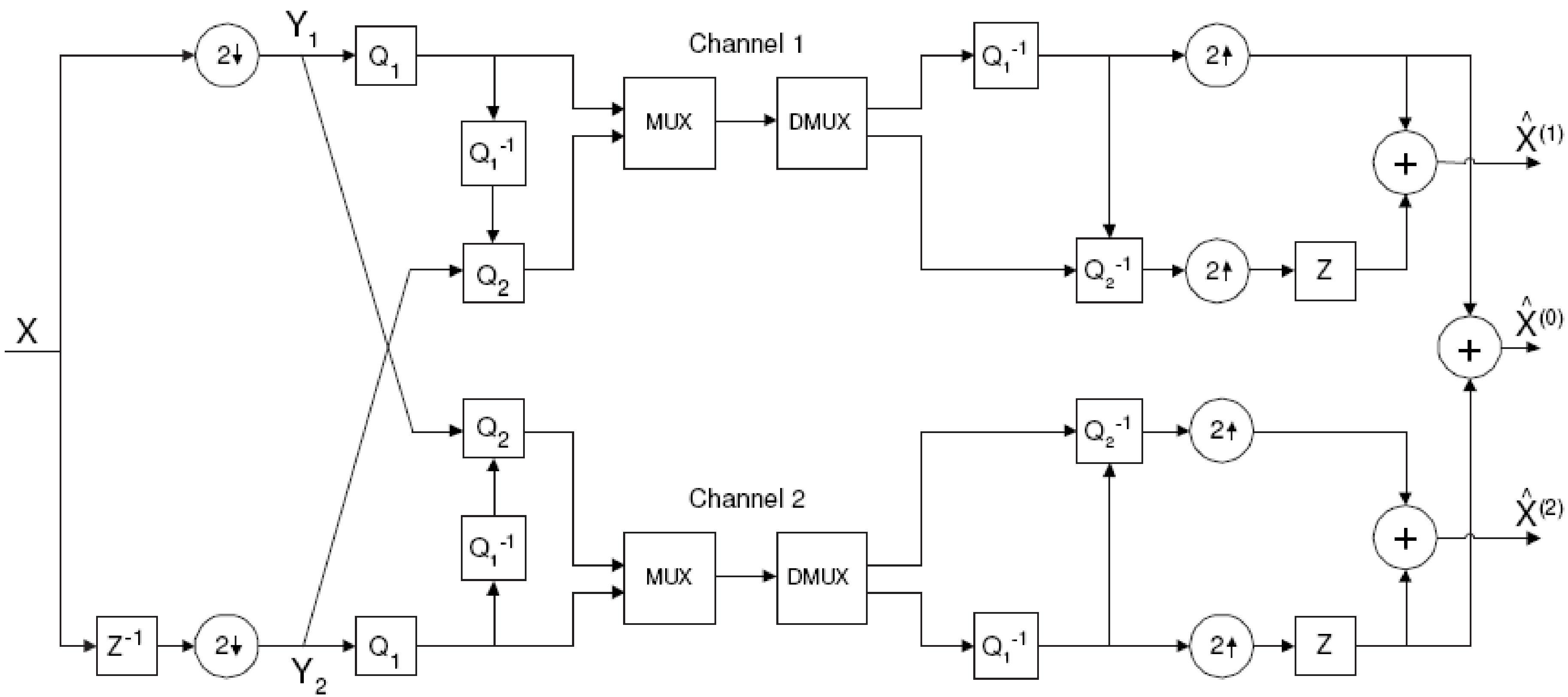
Wavelet Background: Statistical Characterization of Wavelet Coeffs (cont.)

Spatial and scale-to-scale dependencies:

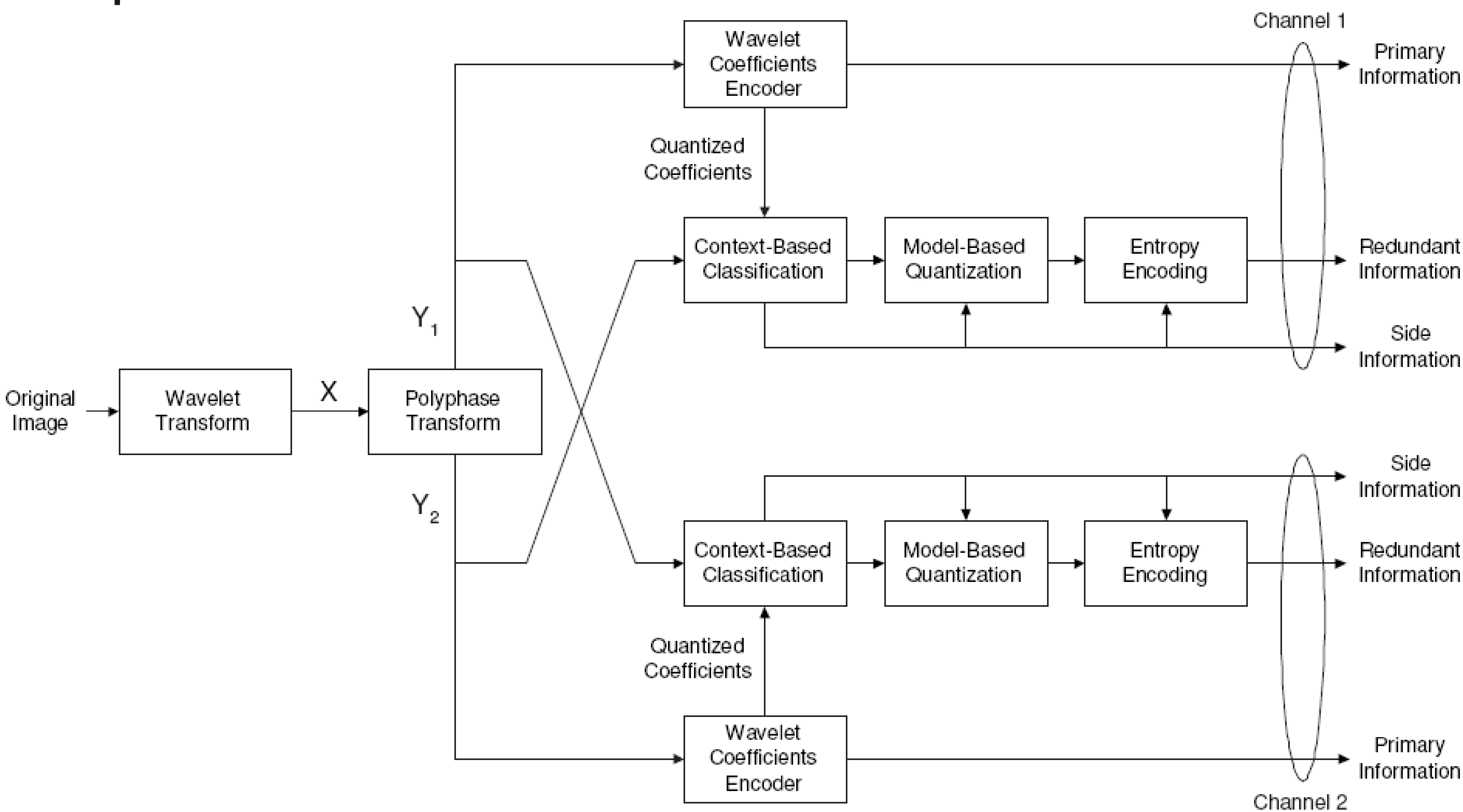
- Wavelet coefficients are not statistically independent (although approximately decorrelated)
- Dependencies are implicitly utilized by numerous image compression schemes (e.g., EZW, SPIHT)

Context-Based MD Wavelet Image Coding: Proposed System Outline

- Concept: Improve coding efficiency of Q_2 via utilization of contextual information from Q_1

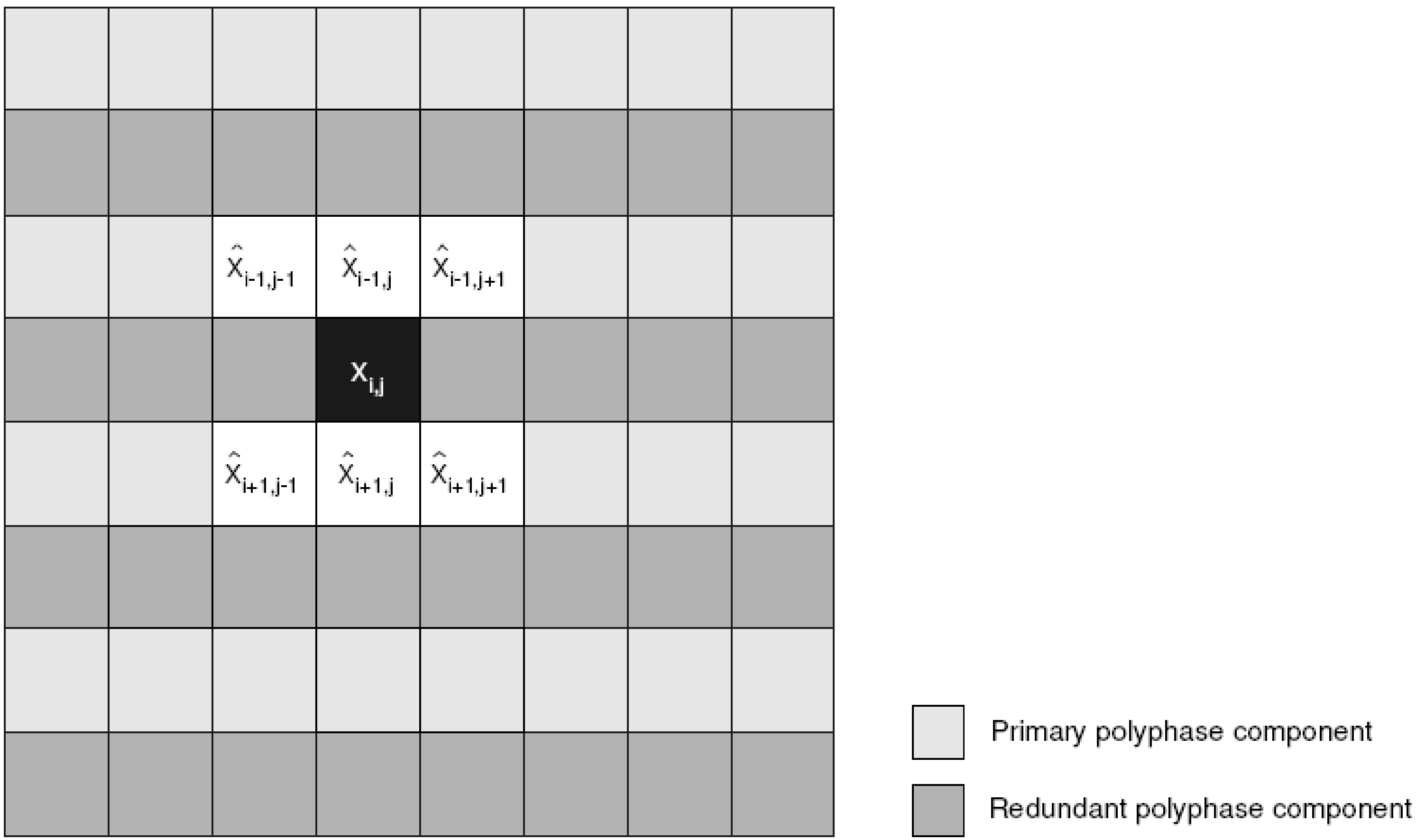


Context-Based MD Wavelet Image Coding: Proposed MD Encoder – Block Diagram



Context-Based MD Wavelet Image Coding: Context Formation

- Classification of the wavelet coefficient $X_{i,j}$ is based on the following context $C_{i,j}$ of quantized local neighbors:



Context-Based MD Wavelet Image Coding: Context-Based Classification

- Classification offers a potential increase in coding efficiency (quantization is adapted to the data)
- Side information is transmitted for improved performance:
  - Classification thresholds
  - Source statistics of each class
- Classification procedure inspired by [Yoo et al., 1999] (SD subband image coder)

Context-Based MD Wavelet Image Coding: Classification Rule

- Purpose: Assign one of a finite number of classes to a coefficient $X_{i,j}$ given its context $C_{i,j}$

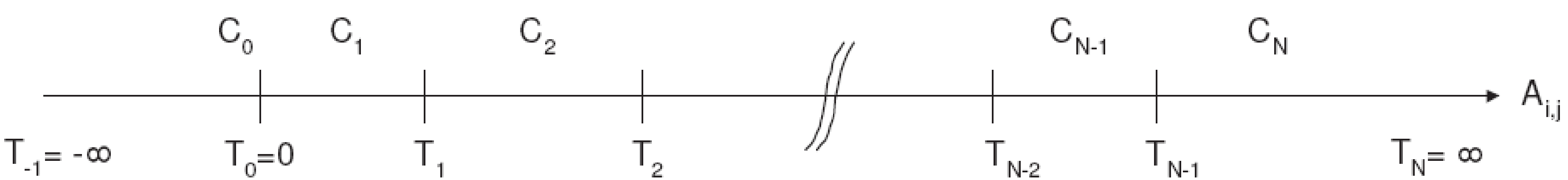
- Classification is based on a weighted average of the magnitudes of coefficients in $C_{i,j}$ ("Activity"):

$$A_{i,j} = a_1|\hat{X}_{i-1,j-1}| + a_2|\hat{X}_{i-1,j}| + a_3|\hat{X}_{i-1,j+1}| + a_4|\hat{X}_{i+1,j-1}| + a_5|\hat{X}_{i+1,j}| + a_6|\hat{X}_{i+1,j+1}|$$

where $\sum_k a_k = 1$

  - E.g.: Weights are inversely proportional to the geometric (Euclidean) distances of the corresponding coeffs in $C_{i,j}$ from $X_{i,j}$

- Classification rule (for set classification thresholds):



Context-Based MD Wavelet Image Coding: Classification Thresholds Design

- Purpose (for a given subband):
  Maximization of the "classification gain" (coding gain due to classification, under certain simplifying assumptions)
- Model assumption: Coeffs in each class of each subband are drawn from a (zero-mean) Laplacian distribution

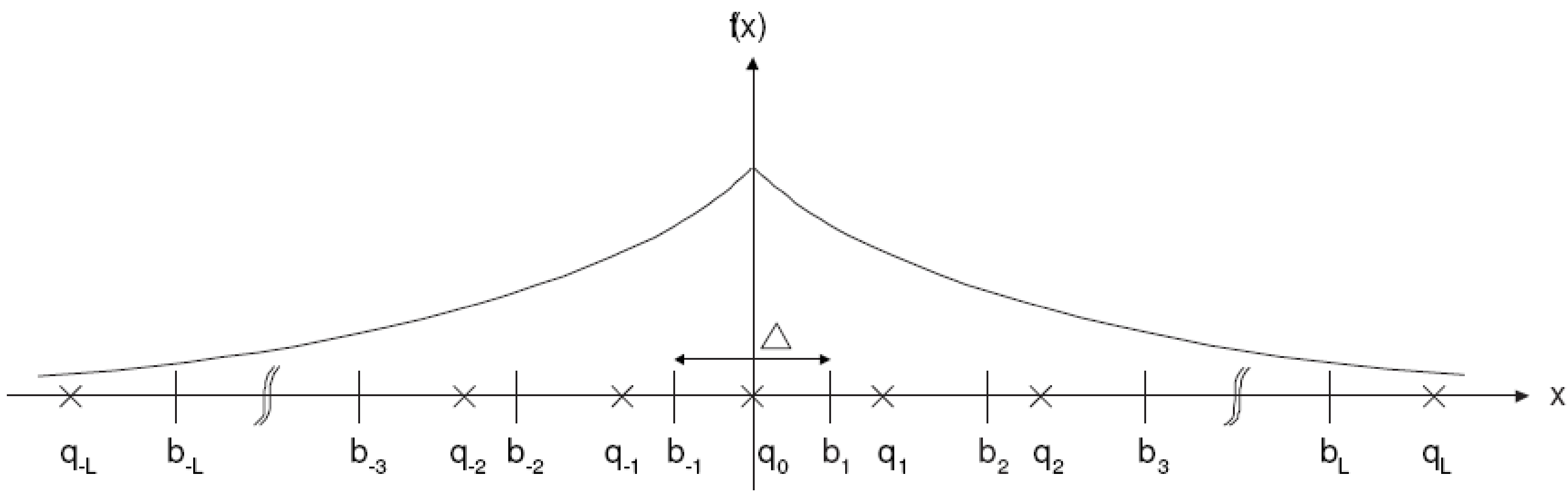
$$f_\lambda(x) = \frac{\lambda}{2} e^{-\lambda|x|}$$

Context-Based MD Wavelet Image Coding: Model-Based Adaptive Quantization

- Purpose: Efficient quantization using a set of quantizers, each customized to an individual class
- Two types of quantizers are examined:
  - Uniform Threshold Quantizer (UTQ)
  - Uniform Reconstruction with Unity Ratio Quantizer (URURQ)
- Both types of quantizers well approximate the optimum ECSQ for the Laplacian distribution (with MSE distortion)
- Both are completely defined by a single parameter Δ

Context-Based MD Wavelet Image Coding: Uniform Threshold Quantizer (UTQ)

- Completely defined by its step size Δ
- Reconstruction levels are optimized for minimum distortion (centroid condition)



Context-Based MD Wavelet Image Coding: Design Strategy for the Quantizers (UTQs)

- Purpose: Avoid complex entropy-constrained design algorithms for the UTQs
- Means: Optimal bit allocation scheme based on a pre-designed array of MSE-optimized UTQs of different step sizes (with no constraint on output entropy)

$\Rightarrow$ Goal: Design an MSE-optimal UTQ with step size Δ for the Laplacian distribution with parameter λ

  - Expressions for bin boundaries, reconstruction levels and bin probabilities are derived straightforwardly (also found in the literature)

Context-Based MD Wavelet Image Coding: Quantizer Function of UTQ for Laplacian Distribution

- Purpose: Estimate rate and distortion of UTQ to obtain its operational DR function (quantizer function)
  - Quantizer function is required for bit allocation
- Rate *R* is estimated by the output entropy of UTQ:

$$H_Q = -\sum_{j=-L}^{j=L} p_j \log_2 p_j$$

- We derive a closed form expression for the distortion *D* :

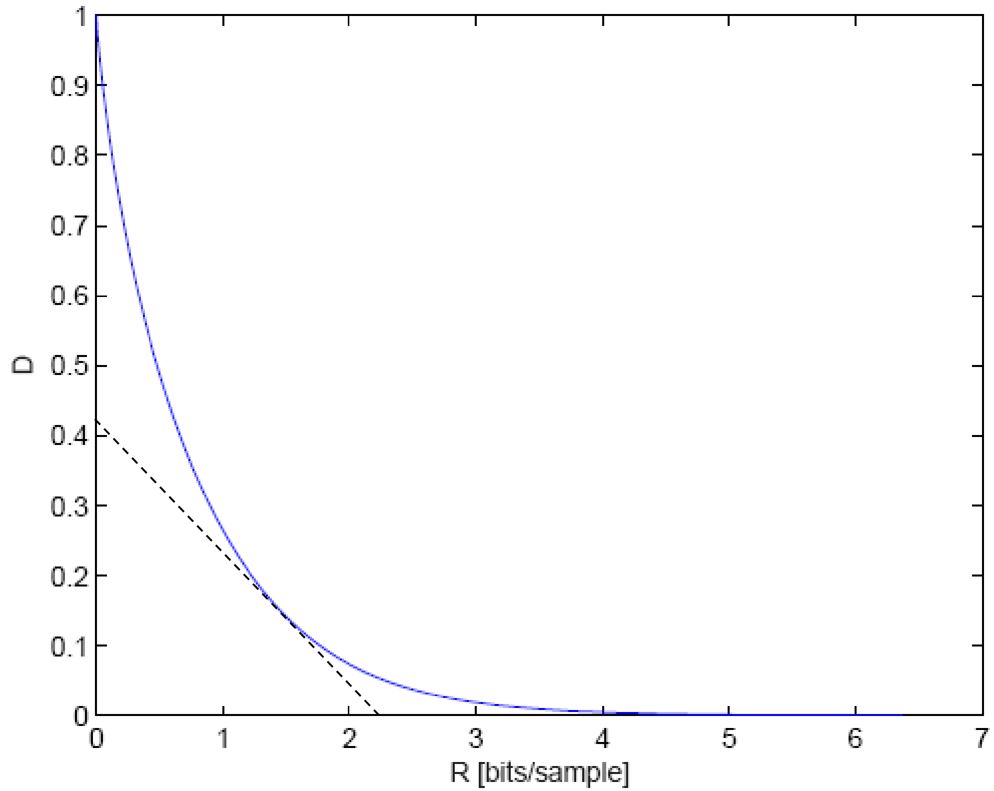
$$\begin{aligned} D \; = \; & \frac{2}{\lambda^2} - e^{-\lambda\frac{\Delta}{2}} \left(\left(\frac{\Delta}{2}\right)^2 + \frac{\Delta}{\lambda} + \frac{2}{\lambda^2} \right) + \\ & + \left(\sum_{j=1}^{L-1} e^{-\lambda q_j} \right) \cdot \left[e^{\lambda\delta} \left(\delta^2 - \frac{2\delta}{\lambda} + \frac{2}{\lambda^2} \right) - e^{\lambda(\delta-\Delta)} \left((\delta-\Delta)^2 - \frac{2(\delta-\Delta)}{\lambda} + \frac{2}{\lambda^2} \right) \right] + \\ & + \frac{1}{\lambda^2} e^{1-\lambda q_L} \end{aligned}$$

where $\delta = \frac{1}{\lambda} - \frac{\Delta}{e^{\lambda\Delta} - 1}$

Context-Based MD Wavelet Image Coding: Quantizer Function of UTQ for Laplacian Distribution (cont.)

- Off-line computation: Array of UTQs obtained for closely spaced values of the step size Δ (for the unit-variance Laplacian distribution)

- Result: Indexed operational DR function (indexed by slope)



Context-Based MD Wavelet Image Coding: Optimal Model-Based Bit Allocation

- Purpose:
  Given the desired redundancy rate, determine the rate at which each UTQ operates

Context-Based MD Wavelet Image Coding: Optimal Model-Based Bit Allocation (cont.)

- Optimization problem:

Find the optimal rates $\{R_b\}_{b=1}^{B}$ such that the overall distortion D is minimized, subject to the constraint $R \leq R_T$

- Lagrangian optimization:
  Minimize the Lagrangian cost function $J(\xi) = D + \xi R$
  (for a fixed Lagrange multiplier ξ, to be determined such that $R = R_T$)
- Resulting rate allocation equations (≈ "constant slope" principle):

$$D_b'(R_b) = -\frac{\xi}{G_b}, \;\; b = 1, \ldots, B$$

(G_b is the synthesis energy gain factor of class *b*'s subband)

  - Solved efficiently via variance scaling (slope normalization)

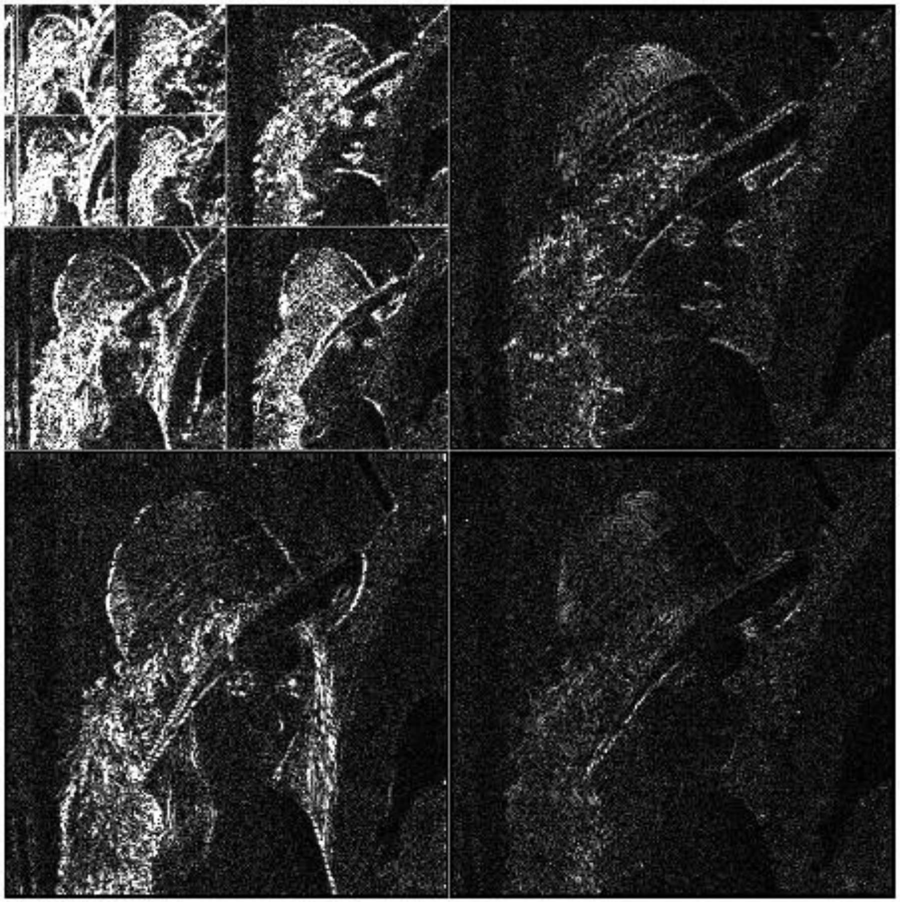
Experimental Results: Configuration

- Wavelet transform:
  - Biorthogonal wavelet transform using Cohen-Daubechies-Feauveau (CDF) 9/7-tap wavelet filters (three-level decomposition)
  - Whole-sample symmetric boundary extension

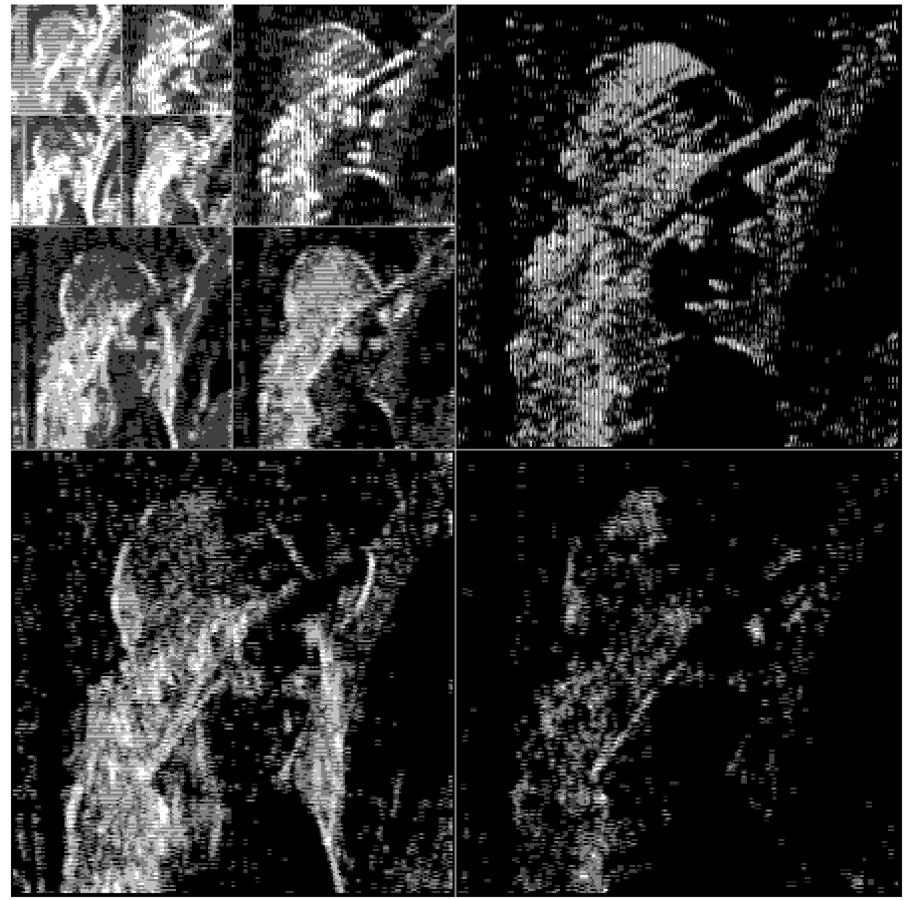
⇒ No coefficient expansion

- For demonstrations: Lena image (grayscale), 512x512 pixels

Experimental Results: Quality of Classification (Subjective)



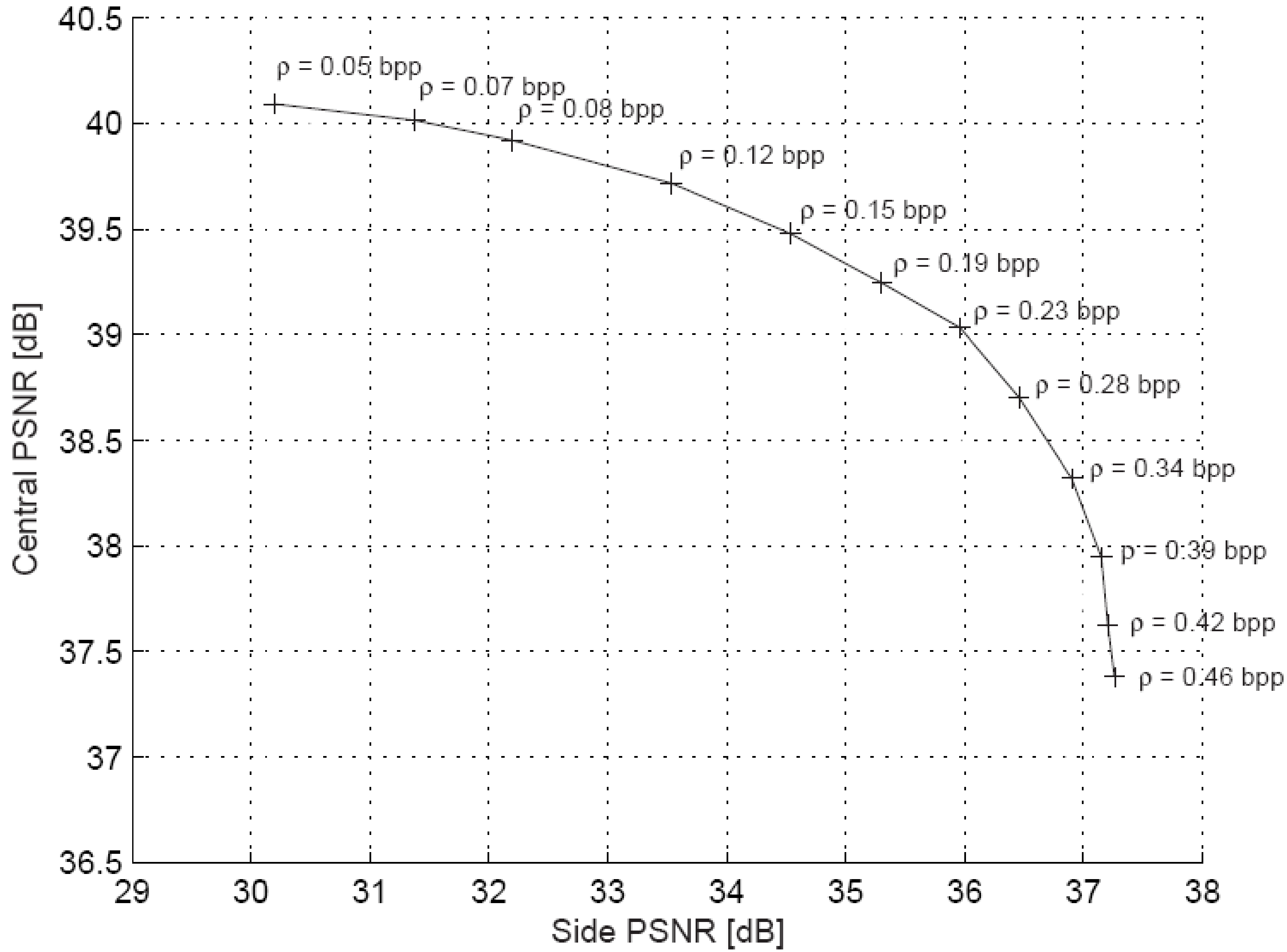
Wavelet transform
(differential approximation)



Classification map

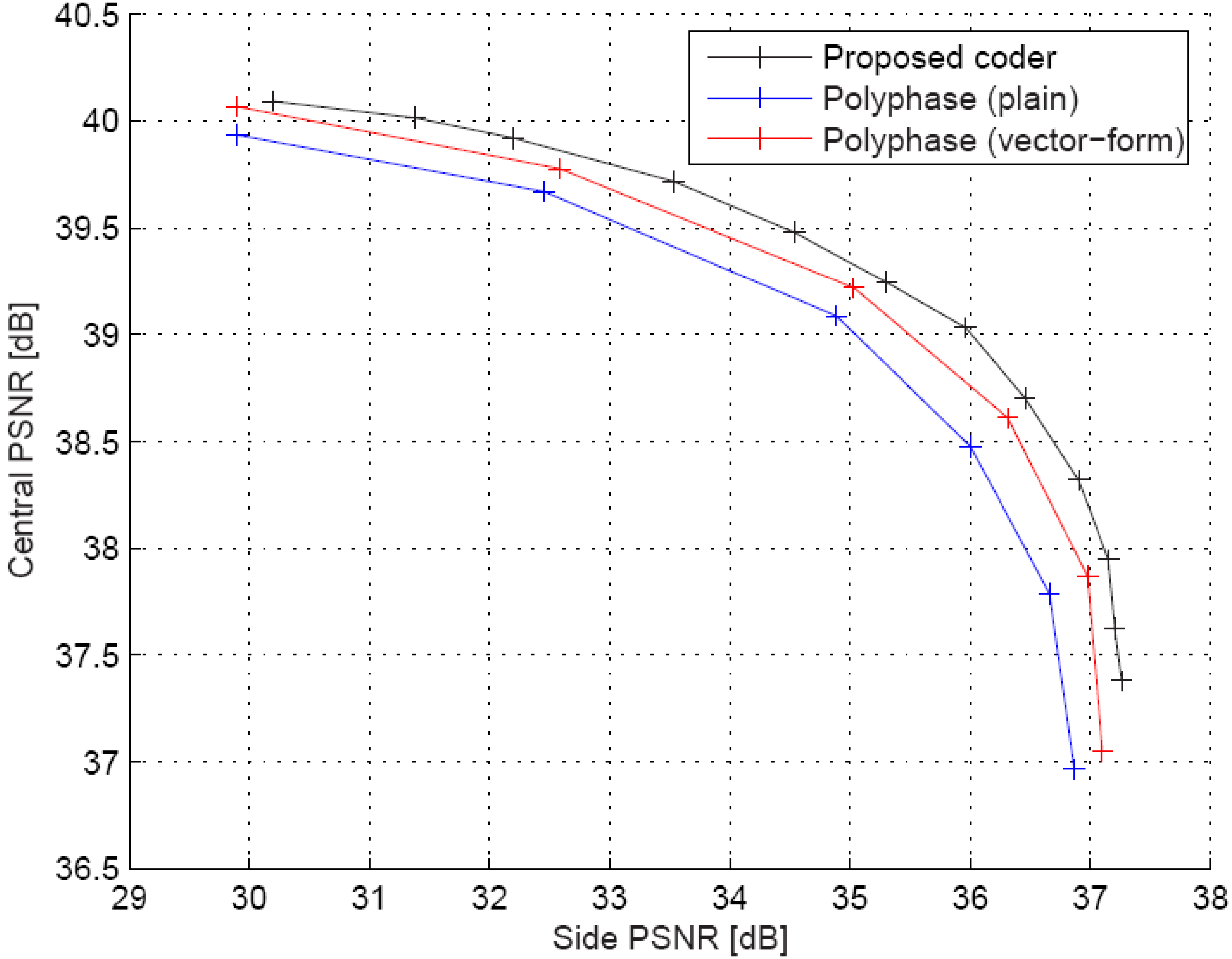
Experimental Results: Performance of the Proposed Coder

Total rate: 1 bpp



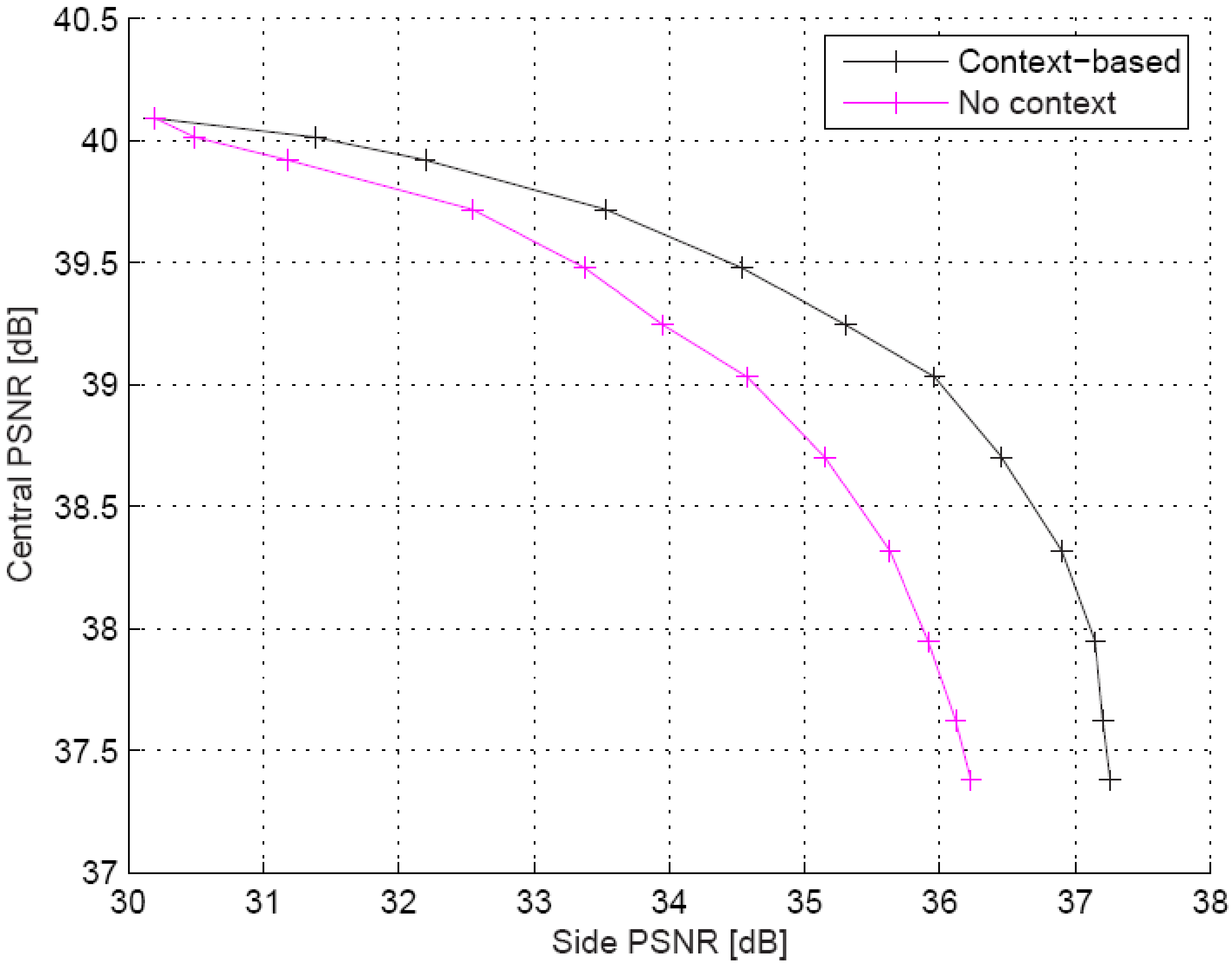
Experimental Results: Performance Compared to Framework Coder

Total rate:
1 bpp



Experimental Results: Context Gain

Total rate:
1 bpp



Summary

- Introduction and fundamentals of MD coding
- Framework: MD coding via polyphase transform
- Proposed context-based MD wavelet image coder
  - Context formation
  - Context-based classficiation
  - Model-based adaptive quantization
  - Optimal model-based bit allocation
- Experimental results
  - Classification results
  - RD performance (also compared to framework)
  - Context gain

Thank You!