2D Object Description And Classification Based On Contour Matching By Implicit Polynomials

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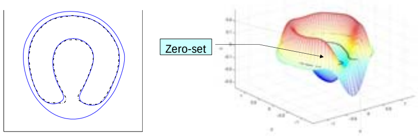
1. Implicit Polynomials

- An implicit polynomial (IP) of degree r:

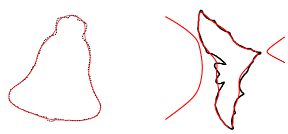
$$P_{\underline{a}}(x,y) = a_{0,0} + a_{1,0}x + a_{0,1}y + a_{2,0}x^2 + a_{1,1}xy + \ldots + a_{0,r}y^r = 0$$

- Alternative way to define IP
  - Intersection of the 3D polynomial with the zero-plane - zero-sets

$$z = a_{0,0} + a_{1,0}x + a_{0,1}y + a_{2,0}x^2 + \ldots + a_{0,r}y^r$$
$$z = 0$$



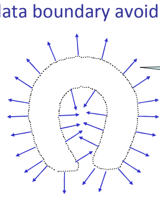
- Modeling
  - describe curves representing the boundary of 2D objects (data-sets) by Zero-sets of IP.



- Classification
  - Represent a boundary of an object by a set of invariants based on IP vector of coefficients $\underline{a}$

2. Gradient-one, MinMax and MinVar – algorithms for object fitting

- Minimize the polynomial values at the data-set points
- Require the gradient to be locally perpendicular to the data boundary avoid zero-set splitting.



Gradient - one

$$\|\nabla P_{\underline{a}}(x,y)\| = 1$$

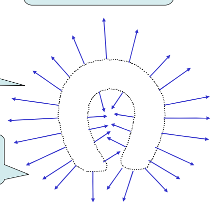
$$\min_{\underline{a}} \sum_{k=1}^{n} P_{\underline{a}}^{r}(x_k,y_k)^2$$

Min-Max

$$\|\nabla P_{\underline{a}}(x,y)\| = \sum_{k=0}^{r}\sum_{i=0}^{k} |x^i y^{k-i}|$$

Min-Var

$$\|\nabla P_{\underline{a}}(x,y)\| = \sqrt{\sum_{k=0}^{r}\sum_{i=0}^{k} x^{2i} y^{2(k-i)}}$$



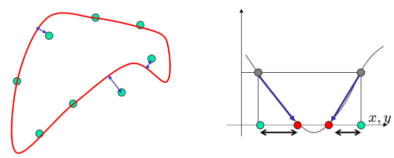
- Properties
  - Min-Max and Min-Var (Helzer at el., 2004) reduce the sensitivity of the zero-set to polynomial coefficients perturbations.
  - Gradient-one (T. Tasdizen at el., 2000) is rotation invariant, while Min-Max and Min-Var are not.

3. Min-Max & Min-Var Modifications

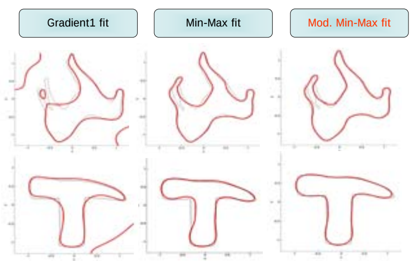
3.1 Improving Fitting Performance

- Minimize approximated **geometric distances** instead of polynomial values

$$\min_{\underline{a}} \sum_{k=1}^{n} P_{\underline{a}}^{r}(x_k,y_k)^2 \;\curvearrowright\; \min_{\underline{a}} \sum_{k=1}^{n} \left(\frac{|P_{\underline{a}}(x_k,y_k)|}{|\nabla P_{\underline{a}}(x_k,y_k)|}\right)^2$$



- Examples



3.2 Invariance to rotation

- Require the gradient value to be dependent only on the distance of the data-set points from the origin
- Solution for Min-Max

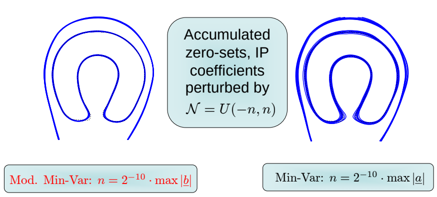
$$\sum_{l=1}^{r}\sum_{i=1}^{l} |x_k^{l-i} y_k^{i}| \Longrightarrow \underset{x^2+y^2=d^2}{mean} \sum_{l=1}^{r}\sum_{i=1}^{l} |x_k^{l-i} y_k^{i}|$$

- Solution for Min-Var
  - redefine the polynomial

$$\mathcal{P}_{\underline{b}}^{\,r}(x,y) = \sum_{l=0}^{r}\sum_{i=0}^{l} b_{l-i,i}\sqrt{\binom{l}{i}}\, x^{l-i}y^{i}$$

$$\sqrt{\sum_{l=1}^{r}\sum_{i=1}^{l} x_k^{2(l-i)} y_k^{2i}} \Longrightarrow \sqrt{\sum_{l=0}^{r}(x^2+y^2)^l}$$

  - Dynamic range of $\underline{b}$ is smaller than the dynamic range of $\underline{a}$ and, hence, $\underline{b}$ is less sensitive to coefficient noise



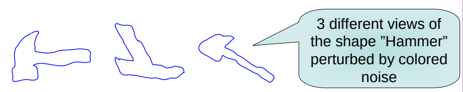
4. Application to 2D Object Recognition

4.1. Data Preprocessing

- Filter out the measurement noise by passing the data-set through a low-pass filter
- Center the data-set and normalize its Scatter Matrix to achieve Affine invariance.
- Scale the data-set so that 75% of it lie inside a circle of radius 1 to avoid large perturbations in the coefficients as a result of data-set noise (T. Tasdizen at el, 2000).

4.2. Multi Order (degree) and Fitting Error Technique (MOFET) for data-set recognition

- **Drawbacks of Single-degree recognition (gen. 6 or 8)**
  - Fails to distinguish contours too complex for selected degree
  - Unstable in case of too simple contours
- **Recognition based on several polynomials**
  - Match polynomials of 4 different degrees (2, 4, 6 and 8) to each of the data-sets and extract the invariants.
  - Use fitting error measure to improve classification performance.
- **Recognition Process**
  - Assume Gaussian PDF of feature vector
  - Learning: estimate the parameters of distribution for each dictionary object.
  - Apply the Maximum Likelihood principle for recognition.
- Experimental data-base
  - "Multiview Curve Database" (MCD) created by M. Zuliani containing 40 shapes X 7 different points of view
  - Each shape perturbed by adding colored noise (20 different perturbations): 40 X 7 X 20 = 5600 objects.



  - 6 views X 20 perturbations (per shape) for dictionary, The remaining sets were used for the recognition test.
- Comparative Results (Classification rate)

MOFET	Single Degree				CSS
	2	4	6	8	
96.6%	69.4%	**94.8%**	94.2%	85.8%	93.8%

5. Conclusion

- **Minimizing approximated geometric distances instead of polynomial values of the data-set, results in improved fitting of the data-set.**
- **Fitting IP of different degrees and utilization of their coefficients and fitting errors benefits from both the stability of low-degree IP and informativeness of high-degree IP, results in a high performance recognizer**