Technion – IIT Dept. of Electrical Engineering

Signal and Image Processing lab

Global Anomaly Detection in Hyperspectral Images via Maximum Orthogonal Complement Analysis (MOCA)

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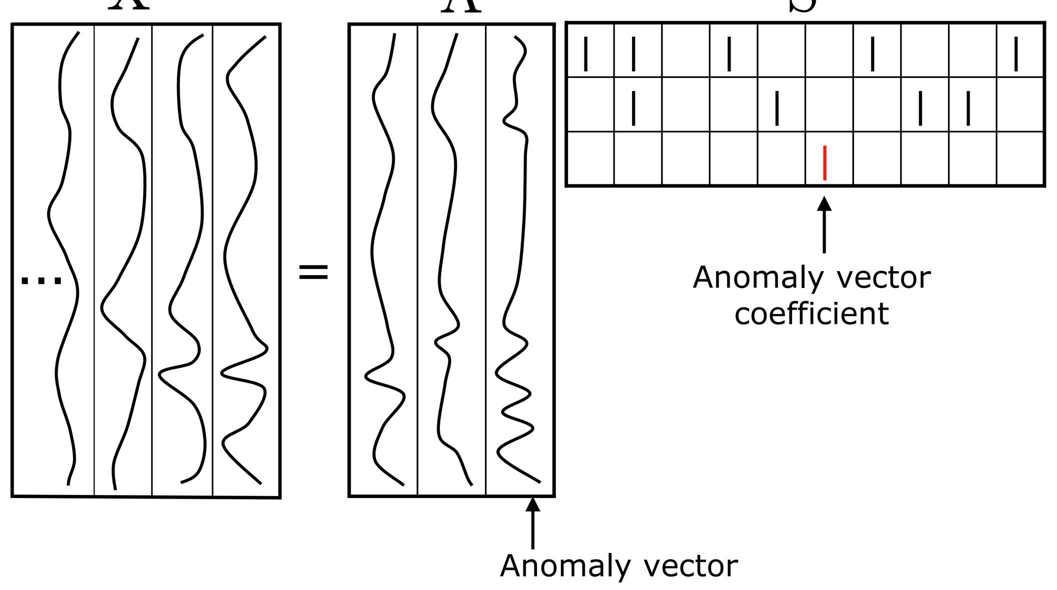
Background versus Anomalies



State of the art approaches: Matched Subspace Detector (MSD)

- Define two hypotheses: $H_0 \;:\; \mathbf{x}_i \sim \mathcal{N}[\mathbf{B}\mathbf{b}_i, \sigma^2\mathbf{I}]$
  $H_1 \;:\; \mathbf{x}_i \sim \mathcal{N}[\mathbf{B}\mathbf{b}_i + \mathbf{T}\theta_i, \sigma^2\mathbf{I}]$
  - $\mathbf{B}$ background subspace basis
  - $\mathbf{T}$ anomaly subspace basis
- Generalized Log-Likelihood Ratio Test (GLRT)

$$L(\mathbf{x}) = \frac{1}{\sigma^2}\mathbf{x}^T\mathcal{P}_{\mathbf{B}^\perp\mathbf{T}}\mathbf{x} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

$\mathcal{P}_{\mathbf{B}^\perp\mathbf{T}}$ is a projection onto $(range\ \mathbf{B})^\perp \bigcap range\ \mathbf{T}$

- Drawbacks
  - The Background and Anomaly subspaces and their ranks are not known

Subspace estimation: ℓ_2-norm based subspace estimation

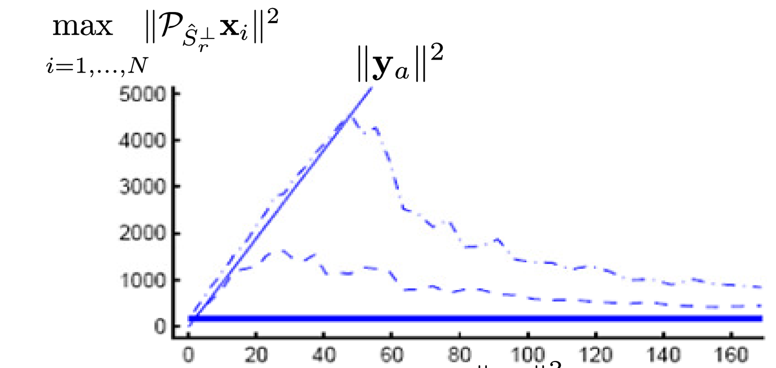
$$\hat{\mathcal{S}}_r = \underset{\mathcal{L}}{\operatorname{argmin}} \|\mathcal{P}_{\mathcal{L}^\perp}\mathbf{X}\|_{Fb}^2 \quad \text{s.t.} \quad \operatorname{rank}\mathcal{L} = r,$$

- The signal subspace $\mathcal{L}$ can be estimated via SVD
- Anomaly vector contributions to the ℓ_2-norm are weaker than contribution of noise
- The resultant subspace is skewed in a way that misses the anomaly vectors with probability close to 1

An example of anomaly misrepresentation as a result of ℓ_2-norm minimization

- Maximum-norm error (dashed and dot-dashed) vs. norm of an anomaly vector (solid)

$N = 10^5, p = 100, r = r_b + r_a = 5 + 1$



$$\mathrm{RSNR} = \frac{\|\mathbf{y}_a\|^2}{(p-r)\sigma^2}$$

Subspace estimation: $\ell_{2,\infty}$-norm based subspace estimation

$$\hat{\mathcal{S}}_r = \underset{\mathcal{L}}{\operatorname{argmin}} \max_{i=1,\dots,N} \|\mathcal{P}_{\mathcal{L}^\perp}\mathbf{x}_i\|^2 = \underset{\mathcal{L}}{\operatorname{argmin}} \|\mathcal{P}_{\mathcal{L}^\perp}\mathbf{X}\|_{2,\infty}^2 \quad \text{s.t.} \quad \operatorname{rank}\mathcal{L} = r,$$

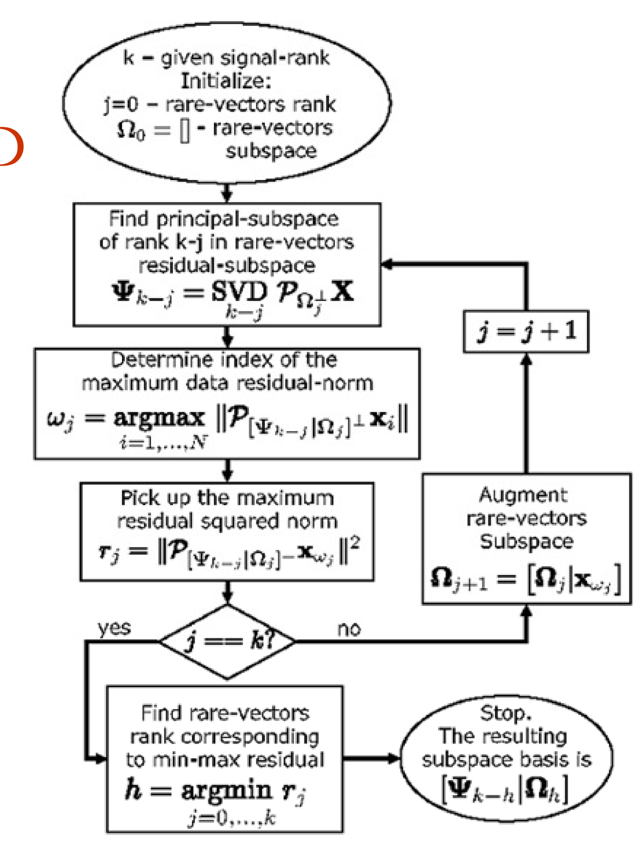
- is equivalent to

$$\hat{\mathcal{S}}_r = \underset{\mathcal{L},\gamma}{\operatorname{argmin}}\ \gamma \quad \text{s.t.} \ \|\mathcal{P}_\perp \mathbf{x}_j\|_2^2 \le \gamma \quad \forall j = 1,\dots,N, \quad \operatorname{rank}\mathcal{L} = r,$$

- very hard to optimize due to a large number of constraints and a non-convex constraint

Greedy MX-SVD

- Look for a basis of the form: $[\mathbf{\Psi}_{k-h}|\mathbf{\Omega}_h]$
- $\mathbf{\Omega}_h$ represents anomaly vectors
- $\mathbf{\Psi}_{k-h}$ represents the background



MX-SVD vs. SVD: Maximum residual norm distribution

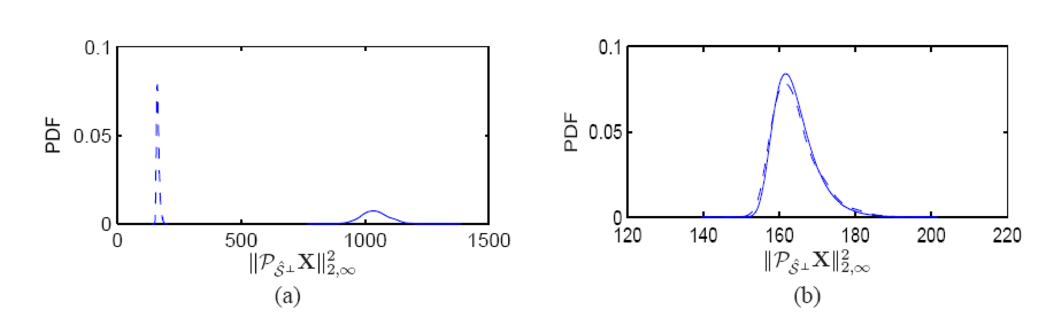
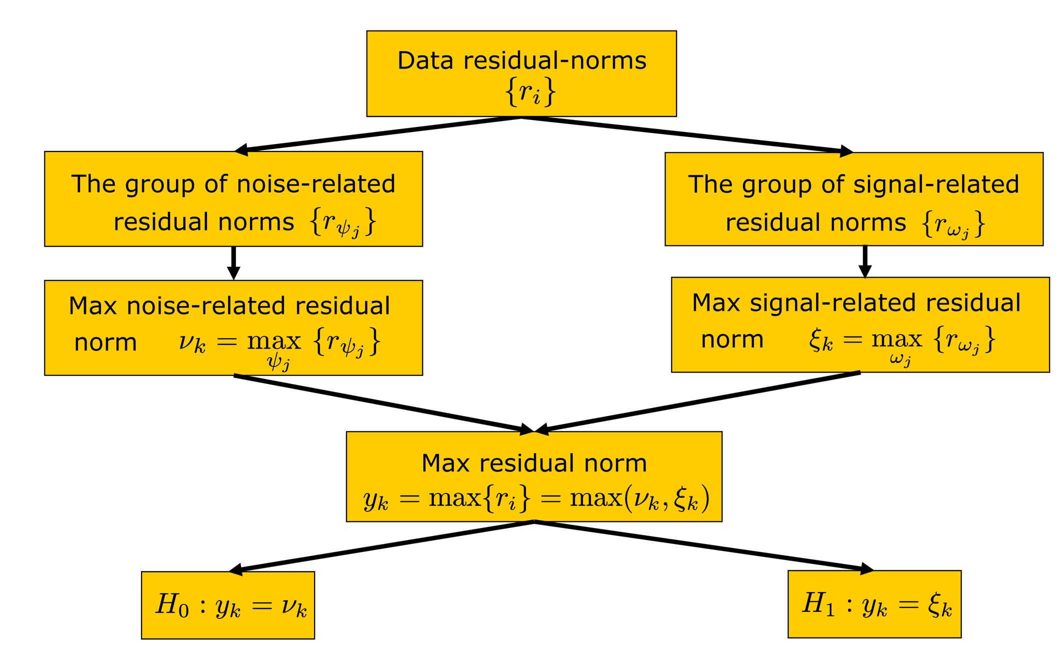


Fig. 4. The pdfs of $\|\mathcal{P}_{\hat{\mathcal{S}}_r^\perp}\mathbf{X}\|_{2,\infty}^2$, obtained via a Monte-Carlo simulation. (a) The empirical pdfs of $\|\mathcal{P}_{\hat{\mathcal{S}}_r^\perp}\mathbf{X}\|_{2,\infty}^2$ obtained by MX-SVD (dashed line) and SVD (solid line) for RSNR = 10, $\sigma = 1, p = 10^2, N = 10^5$, $k = r_{bkgnd} + r_{rare} = 5 + 3 = 8$ (b) The empirical pdf of $\|\mathcal{P}_{\hat{\mathcal{S}}_r^\perp}\mathbf{X}\|_{2,\infty}^2$ by MX-SVD (dashed-line) versus the exact pdf of $\|\mathcal{P}_{\mathcal{S}^\perp}\mathbf{Z}\|_{2,\infty}^2$ (solid line).

Anomaly misrepresentation via ℓ_2

Rank estimation via MOCA

Maximum-residual norm = maximum of 2 maxima

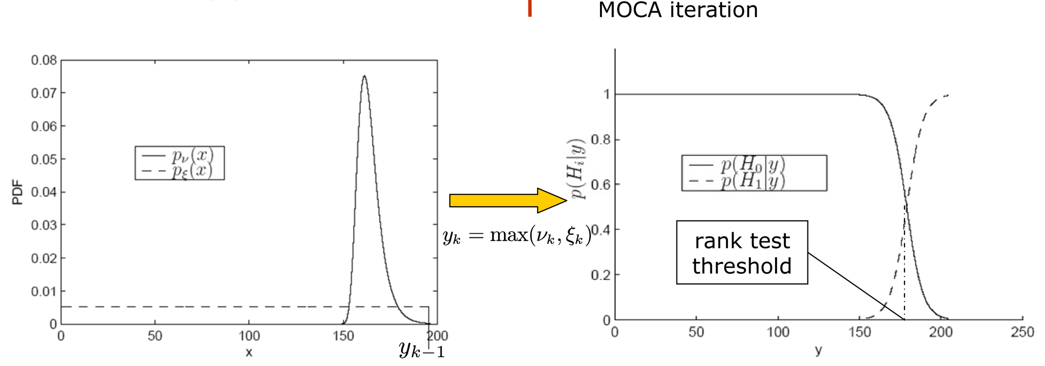


Rank estimation via MOCA

$P(H_0\,|\,y_k)$ and $P(H_1\,|\,y_k)$ are functions of $p_{\nu_k}(\cdot)$ and $p_{\xi_k}(\cdot)$

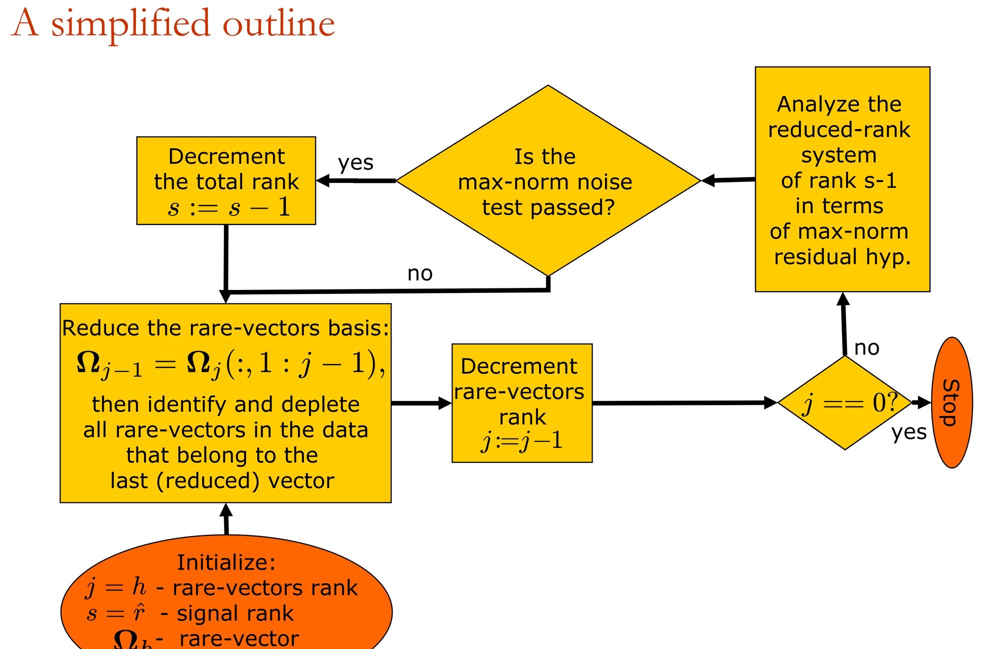
$p_{\nu_k}(\cdot)$ $\qquad$ $p_{\xi_k}(\cdot)$

- Distribution of the maximum noise-norm: $P(\nu_k \le x) = \mathcal{G}(a_N(x - b_N))$
- Gumbel distribution: $\mathcal{G}(x) = e^{-e^{-x}}$
- The maximum signal-norm is assumed to be uniformly-distributed: $\xi_k \sim U(0, y_{k-1})$
- y_{k-1} - is the maxim-norm of data-residuals obtained in the previous MOCA iteration



Anomaly Extraction and Discrimination Algorithm (AXDA)

A simplified outline



ROC curves

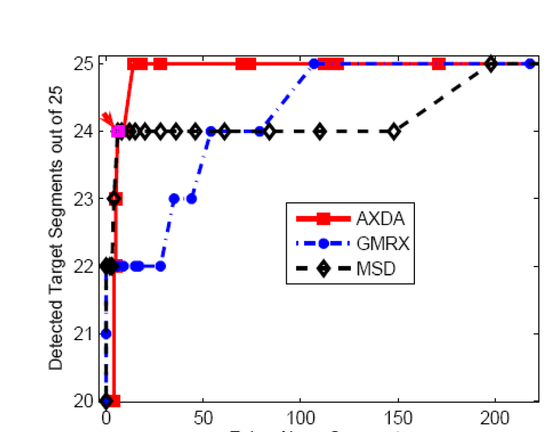
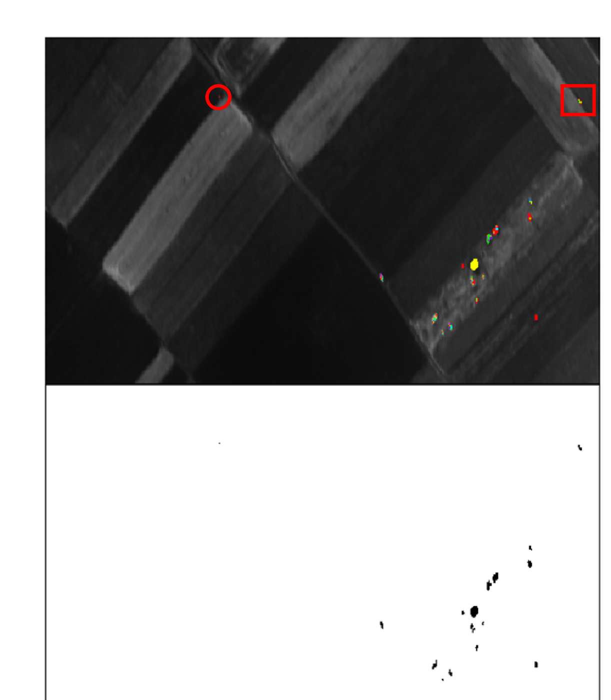


Fig. 6. ROC curves corresponding to GMRX, MSD and AXDA. The nominal operating point of AXDA is marked in magenta color and is pointed out by the arrow. This point corresponds to 24 detected anomalies and 6 false alarm segments.

Real Data Results

$r = 19$, $h = 11$, $r_b = 14$, # anomaly spectra = 11



- r – the estimated signal-subspace rank
- h – the estimated number of rare-vectors
- r_b – the obtained background rank
- # anomaly spectra – the obtained number of anomaly spectra in the scene
- ○ - a single-pixel man-made anomaly
- □ - a vegetation-related anomaly