Technion – IIT Dept. of Electrical Engineering
Signal and Image Processing lab
Global Anomaly Detection in Hyperspectral Images
via Maximum Orthogonal Complement Analysis (MOCA)

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Background versus Anomalies

State of the art approaches:
Matched Subspace Detector (MSD)

- Define two hypotheses:
  \( H_0 : x_i \sim \mathcal{N}(b_i, \sigma^2 I) \)
  \( H_1 : x_i \sim \mathcal{N}(b_i + \mathcal{R}_i, \sigma^2 I) \)

- \( \mathcal{H} \) background subspace basis
- \( \mathcal{T} \) anomaly subspace basis

- Generalized Log-Likelihood Ratio Test (GLRT)
  \( L(x) = \frac{1}{2} \log \frac{p_{H_0}(x)}{p_{H_1}(x)} \)
  \( P_{H_0} \) is a projection onto \( \mathcal{H} \)
  \( P_{H_1} \) is \( \mathcal{T} \)

- \( \mathcal{T} \) is a simplified outline

Subspace estimation
\( \ell_2 \)-norm based subspace estimation

- \( \hat{S}_i = \arg \min_{\mathcal{C}} \| P_{H_0} x_i \|^2 \)
  s.t. \( \text{rank } \mathcal{C} = r_i \)

- \( S_i \) is equivalent to \( \hat{S}_i \)

- Very hard to optimize due to a large number of constraints and a non-convex constraint

Greedy MX-SVD

- Look for a basis of the form:
  \( \mathcal{V}_k \) represents anomaly vectors
  \( \mathcal{V}_k \) represents the background

Subspace estimation
\( \ell_\infty \)-norm based subspace estimation

- \( \hat{S}_i = \arg \min_{\mathcal{C}} \| P_{H_0} x_i \|_\infty \)
  s.t. \( \text{rank } \mathcal{C} = r_i \)

- \( S_i \) is equivalent to \( \hat{S}_i \)

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MX-SVD vs. SVD

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<thead>
<tr>
<th>Metric</th>
<th>MX-SVD</th>
<th>SVD</th>
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<tbody>
<tr>
<td>Max residual norm distribution</td>
<td>( \approx )</td>
<td>( \approx )</td>
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An example of anomaly misrepresentation as a result of \( \ell_2 \)-norm minimization

- Maximum residual error (dashed and dot-dashed) vs. norm of an anomaly vector (solid)
- \( N = 10^9, p = 100, r = r_0 + r_a = 6 + 1 \)

- Decrement of rank \( s-1 \) the total rank

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- Anomaly Extraction and Discrimination Algorithm (AXDA)
  A simplified outline

- ROC curves

- Real Data Results
  - \( r \) the estimated signal rank
  - \( h \) the estimated number of false alarms
  - \( r_a \) the estimated background rank
  - \( h_a \) the estimated false alarm rate
  - A anomaly response – The threshold parameter in the system
  - \( \sigma \) the standard deviation of the anomaly

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