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Department of Electrical Engineering
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Self-dual Morphological Methods Using Tree Representation

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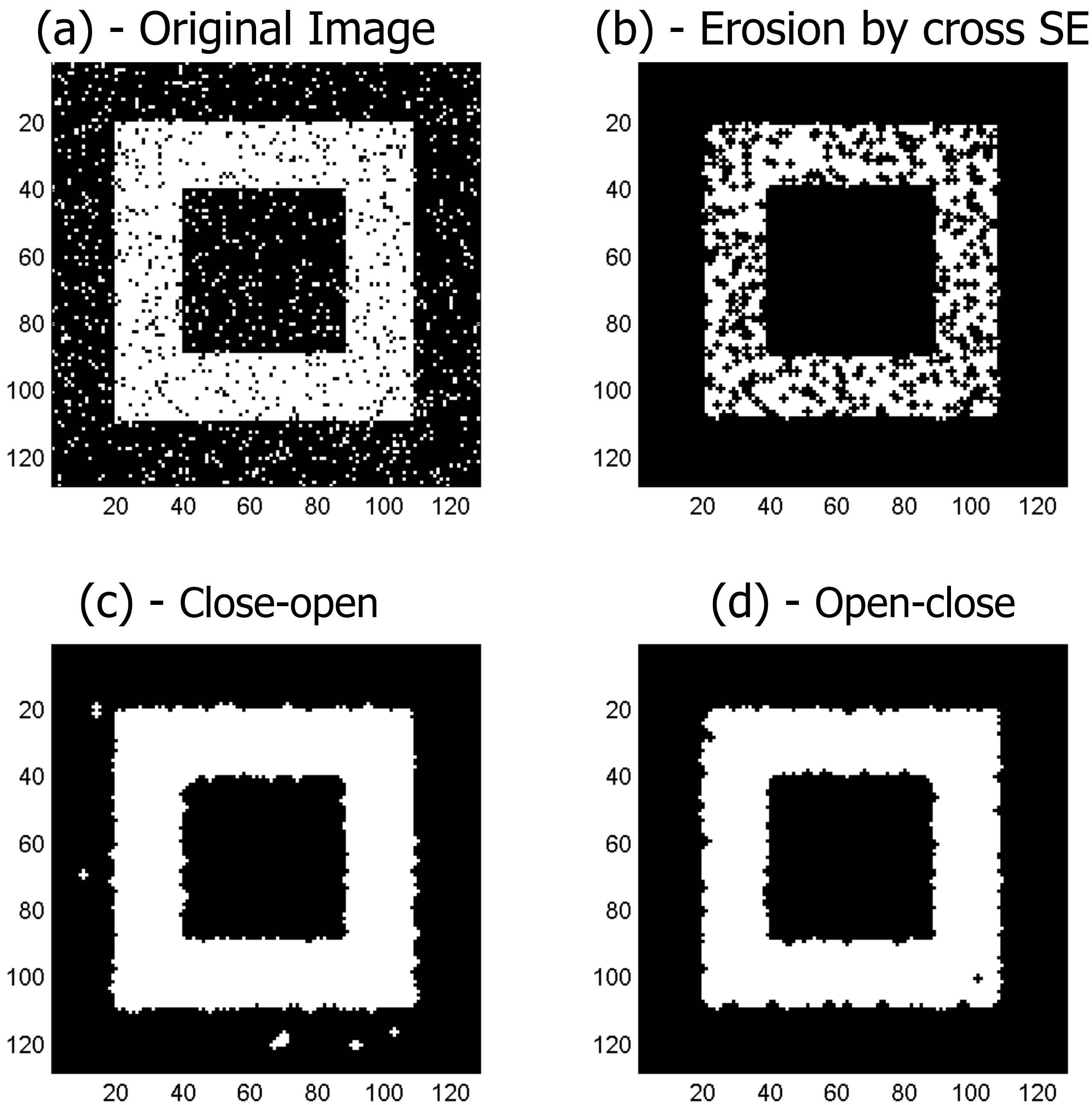
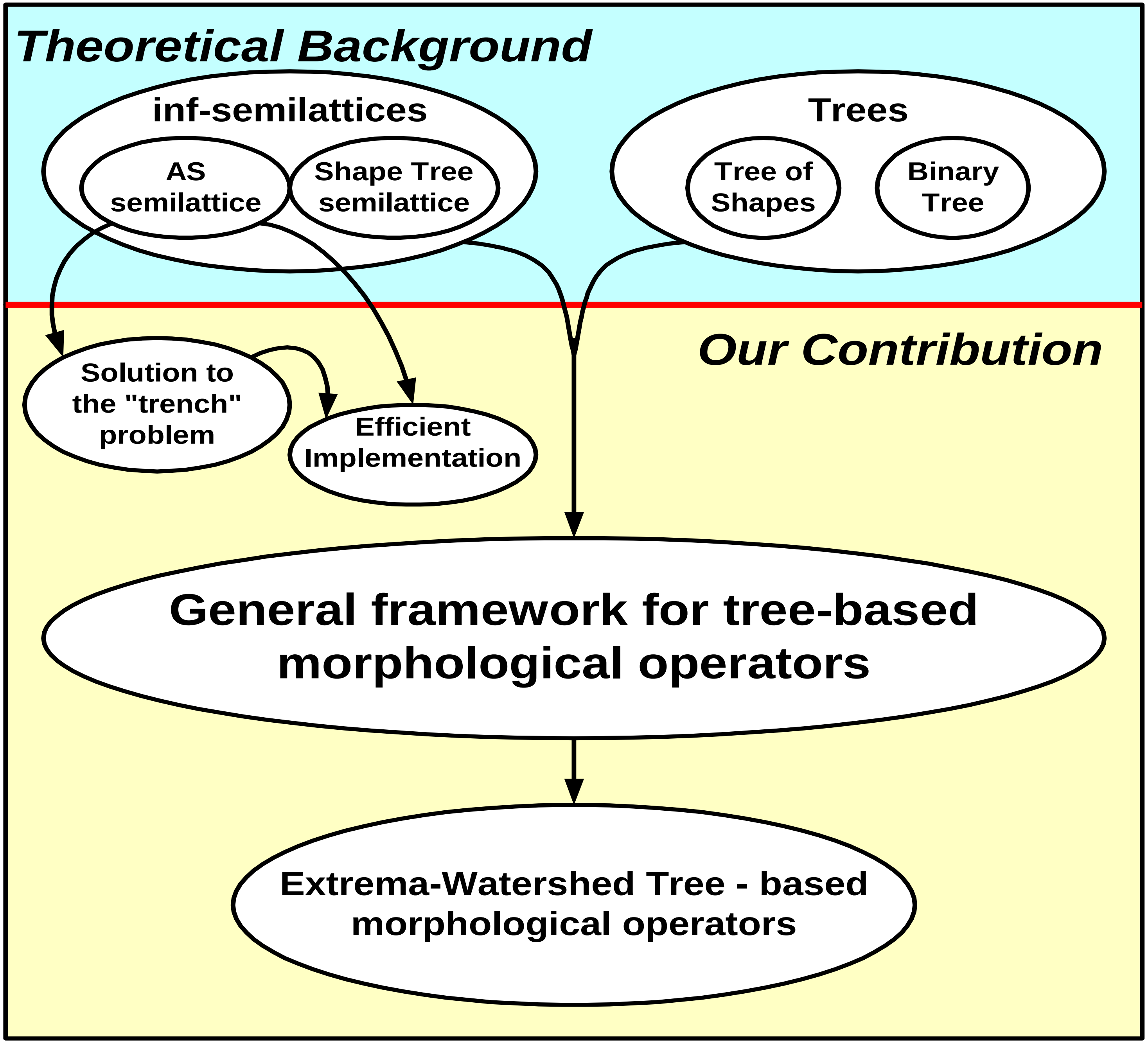
Self-Duality

- Operator ψ is self-dual, if $\psi(f)=-\psi(-f)$.
- Same treatment of dark and light objects.
- Important to many applications, including filtering.
- Linear operators are always self-dual.
- Morphological operators are usually not.


(a) - Original Image
(b) - Erosion by cross SE
(c) - Close-open
(d) - Open-close
20
40
60
80
100
120

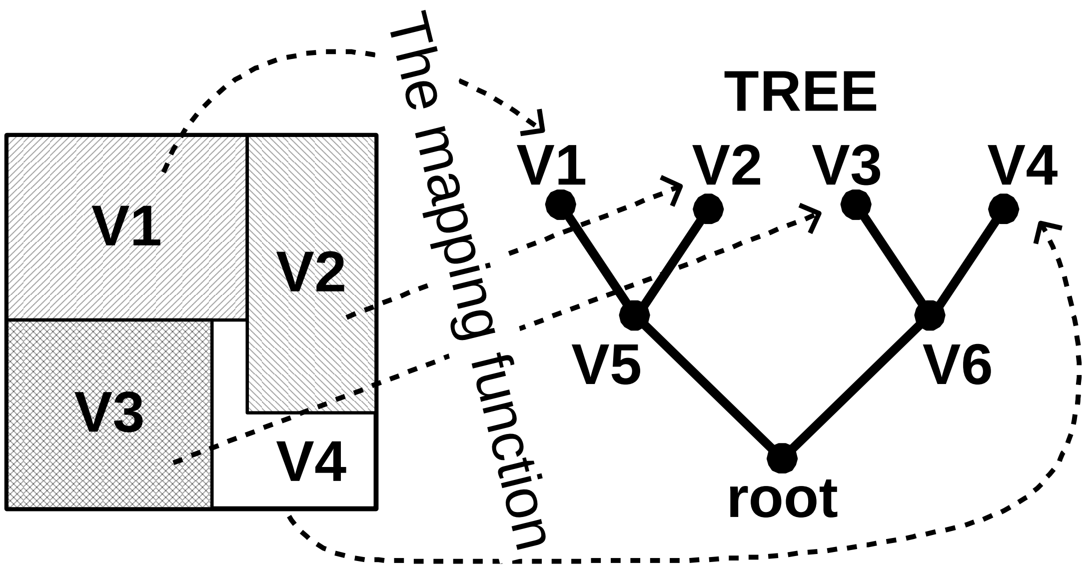
Goals

- This research investigated a family of self-dual morphological operators
  - Developing a new general framework for producing useful morphological operators
  - Examining the framework by a specific usage example
  - Studying the “trenches” phenomena


Theoretical Background
inf-semilattices
AS semilattice
Shape Tree semilattice
Trees
Tree of Shapes
Binary Tree
Our Contribution
Solution to the "trench" problem
Efficient Implementation
General framework for tree-based morphological operators
Extrema-Watershed Tree - based morphological operators

Tree Representation

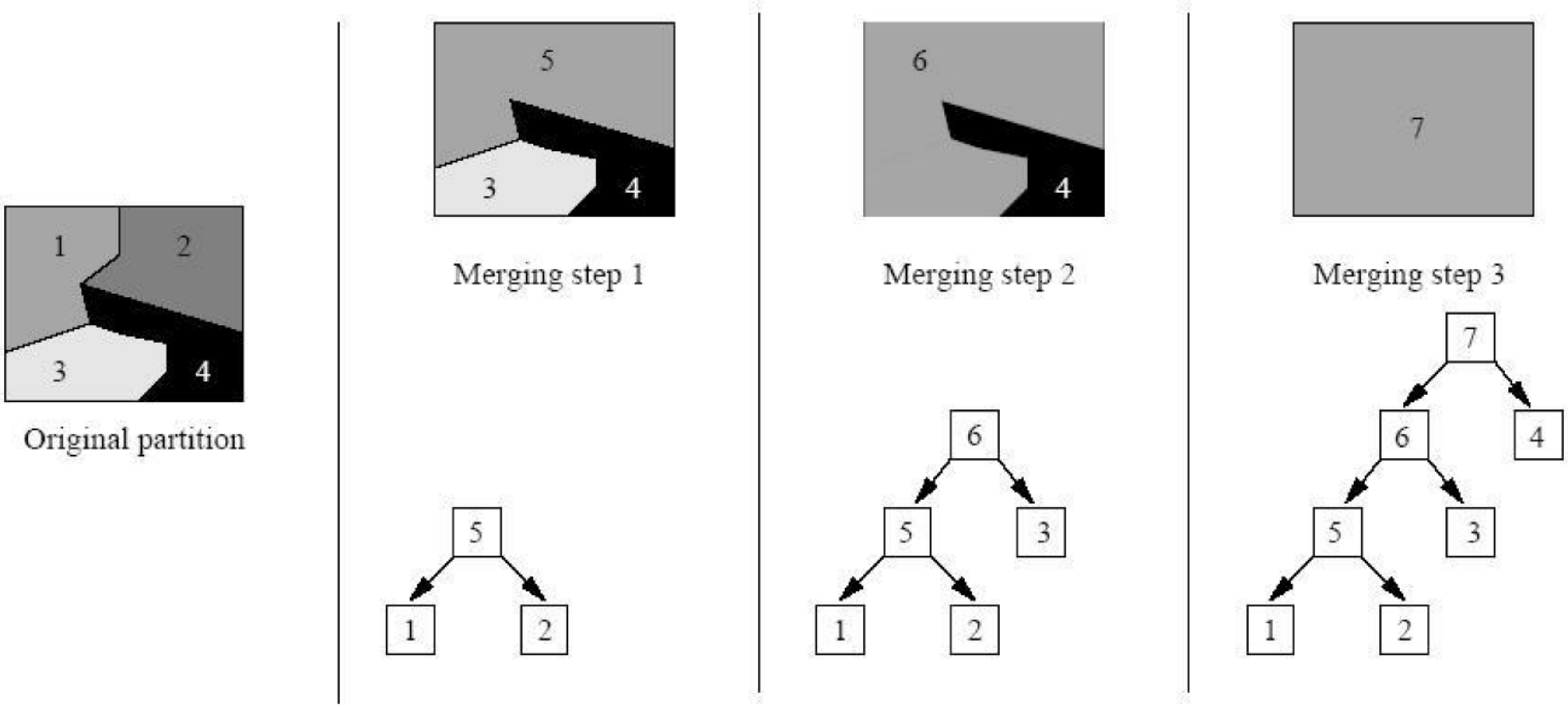
- **Flat zone of an image**
  - A connected region with constant gray-level.
- **Tree Representation**
  - A **tree** and
  - A **mapping function** - maps all flat zones to tree vertices.



Binary partition trees

(P. Salembier and L. Garrido, 2000)

- Are obtained from the partition of the flat zones.
- The leaves of the tree are flat zones of the image.
- The remaining nodes are obtained by merging.
- The root node is the entire image support.



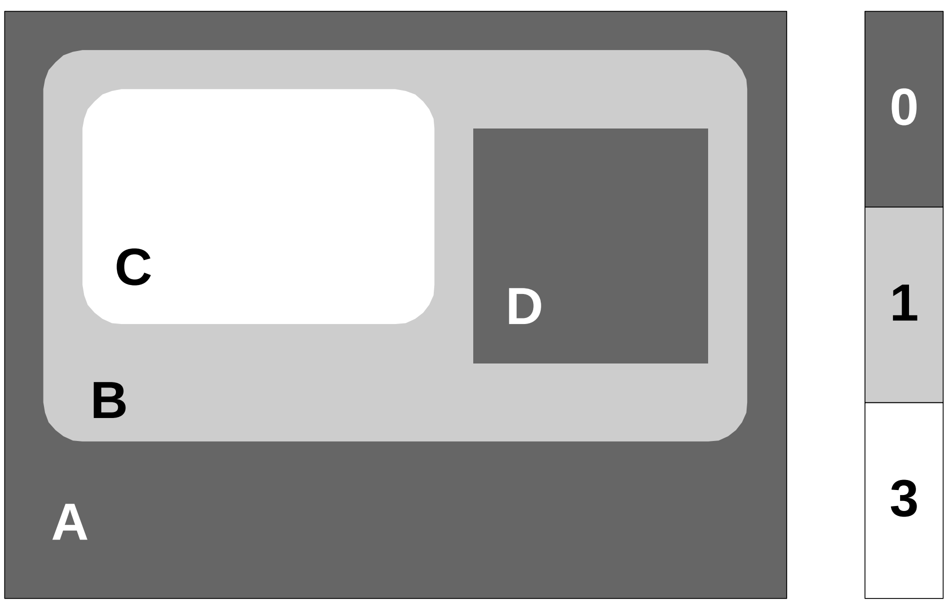
Tree of Shapes (P. Monasse and F. Guichard, 2000)

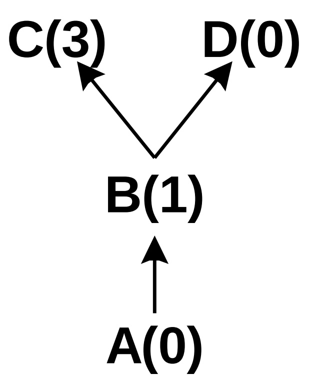
- Represents an image as hierarchy of shapes.
- Build according to the inclusion order.
- Each father vertex area includes also all sons area.
- Self-dual.

$$T_n(f) = \{x \in E \mid f(x) \geq n\}$$

$$FillHole(ConComp(T_n(f)))$$

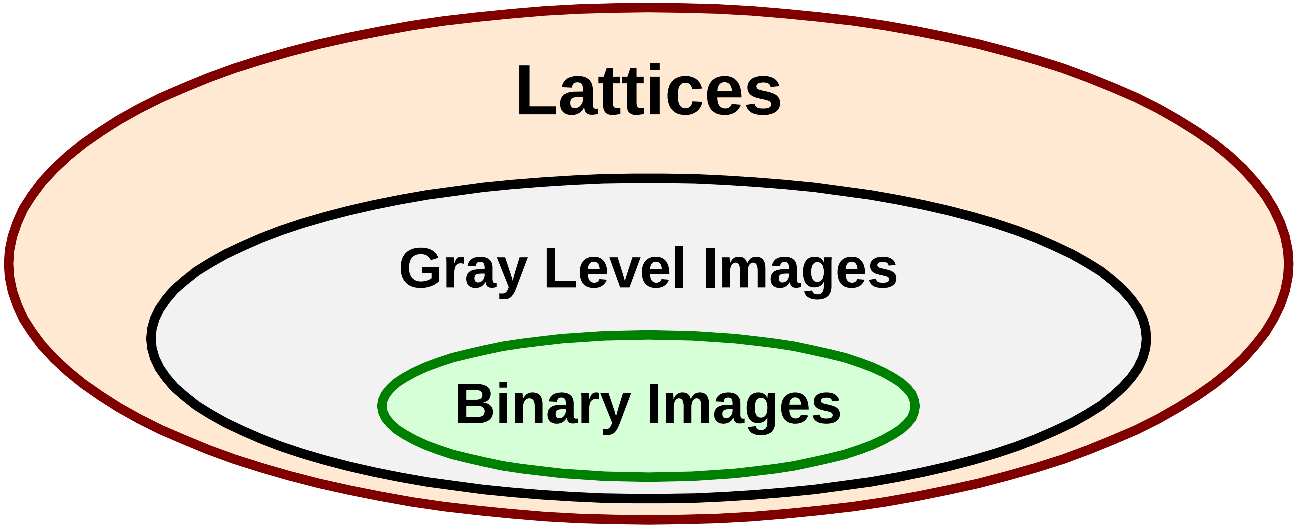
$$FillHole(ConComp(T_n^C(f)))$$





Tree of Shapes

Morphology in Lattices (J. Serra, 1988)



- A *partially ordered set* (poset) P is a set associated with a binary operator ≤.
- A poset L is called a *lattice*, if every subset K, has an infimum ΛK and a supremum VK in L.

Morphological operators in lattices

- An erosion ε :

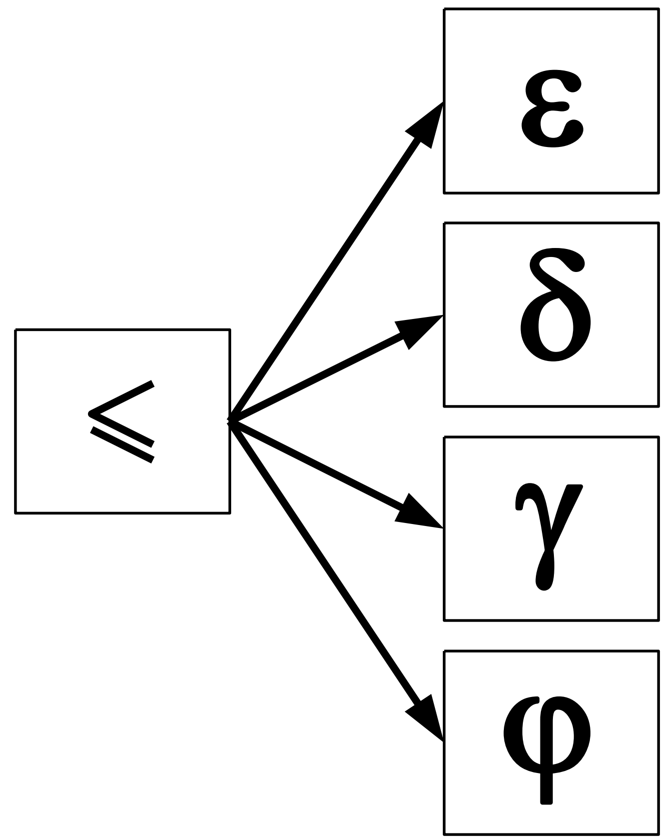
B – Structuring Element (SE)

$$\varepsilon_B(X) = \left(\bigwedge_{i \in B} X_i \right)$$

- A dilation δ :

$$\delta_B(X) = \left(\bigvee_{i \in B} X_i \right)$$

- Opening γ(X)=δε(X),
- Closing φ(X)=εδ(X).

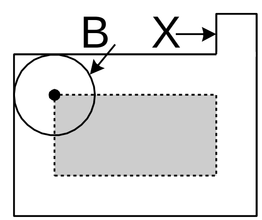


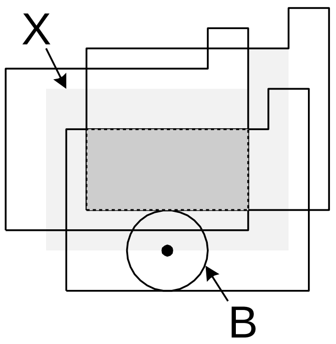
Binary morphological operators

- An erosion ε :

B – Structuring Element (SE)

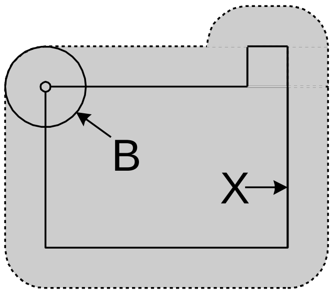
$$\varepsilon_B(X)=\left(\bigcap_{i\in B} X_i\right)$$



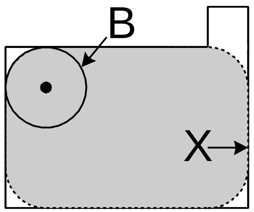


- A dilation δ :

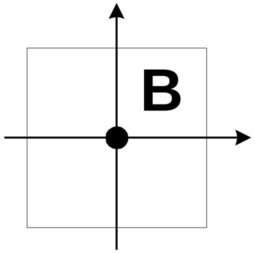
$$\delta_B(X)=\left(\bigcup_{i\in B} X_i\right)$$



- Opening γ(X)=δε(X),
- Closing φ(X)=εδ(X).

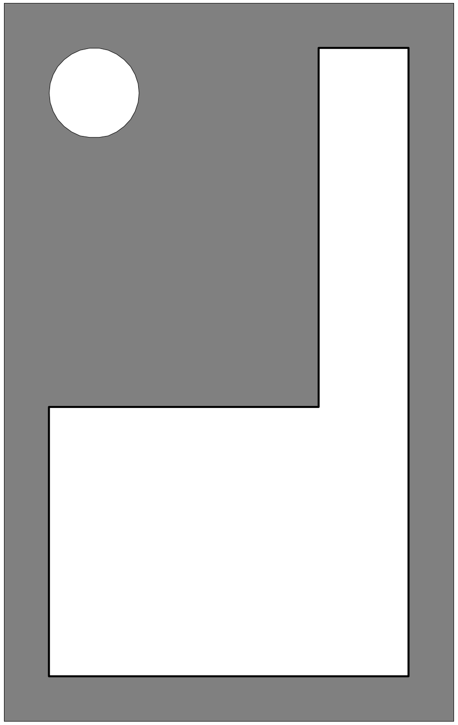


Opening by reconstruction

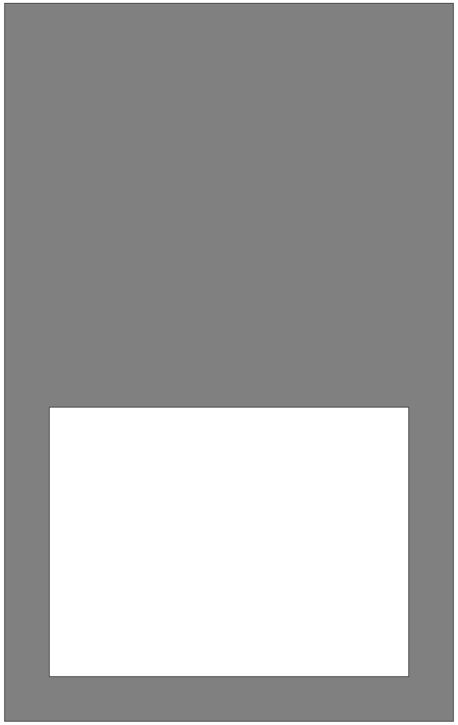


B - Structuring Element (SE)

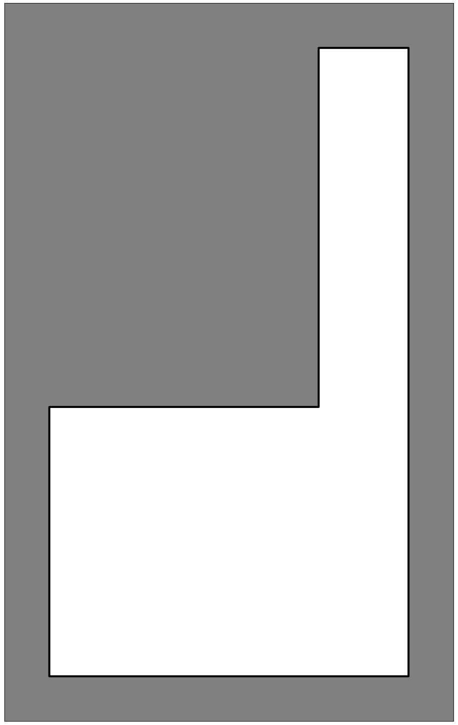
Original image



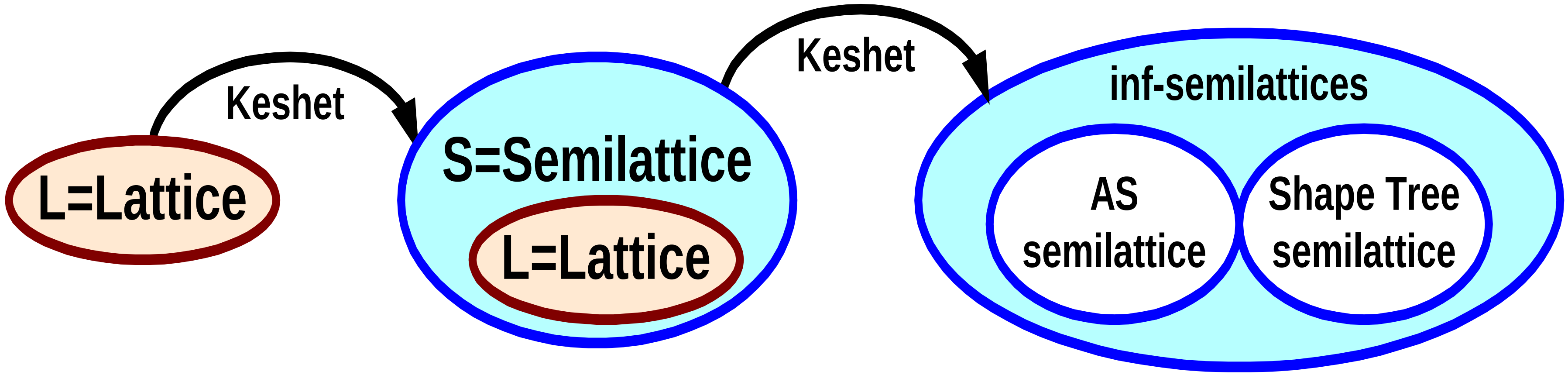
Opened image



Opening by reconstruction

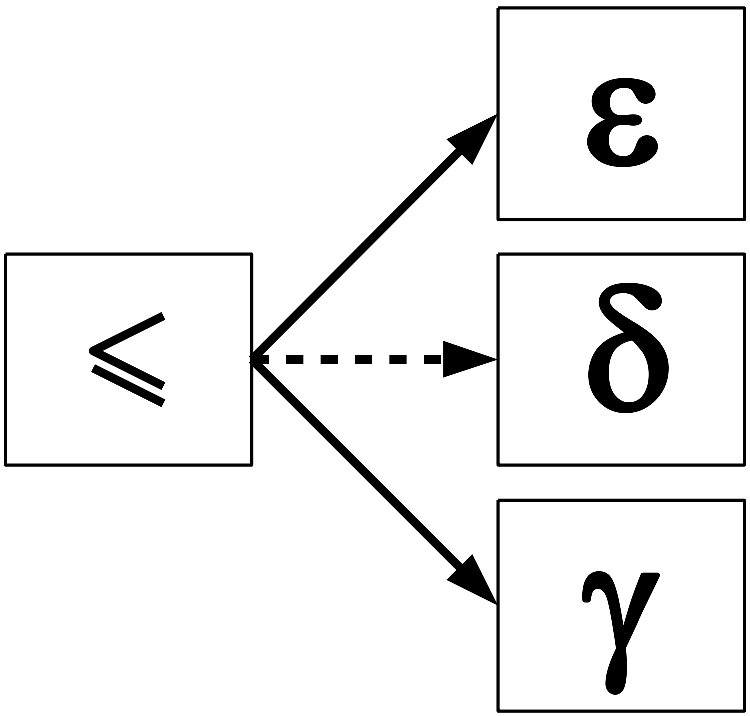


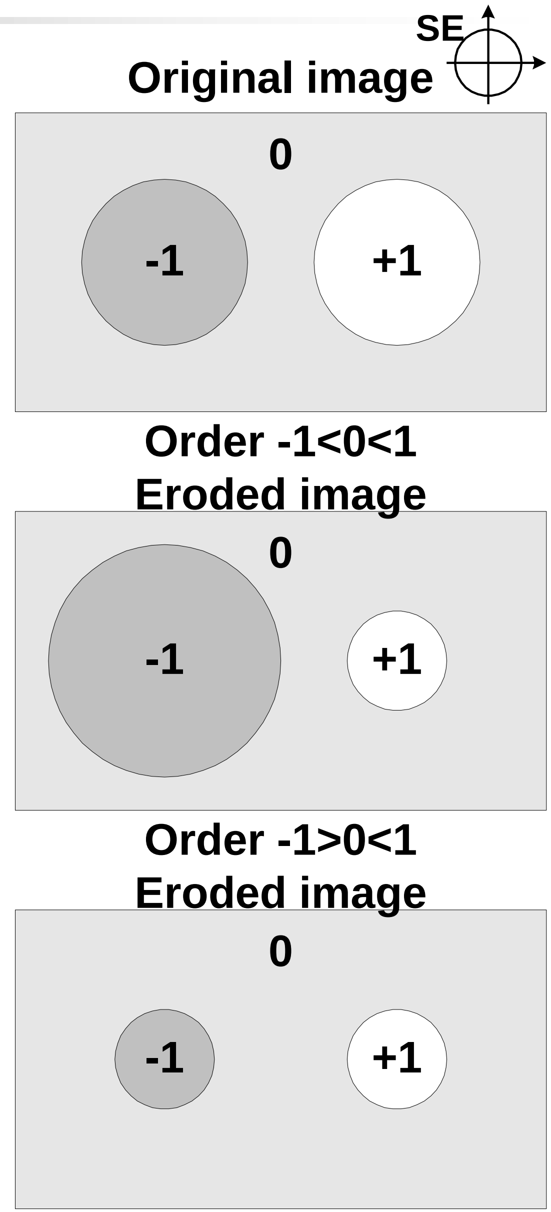
Morphology in Semilattices (R. Keshet, 1998)



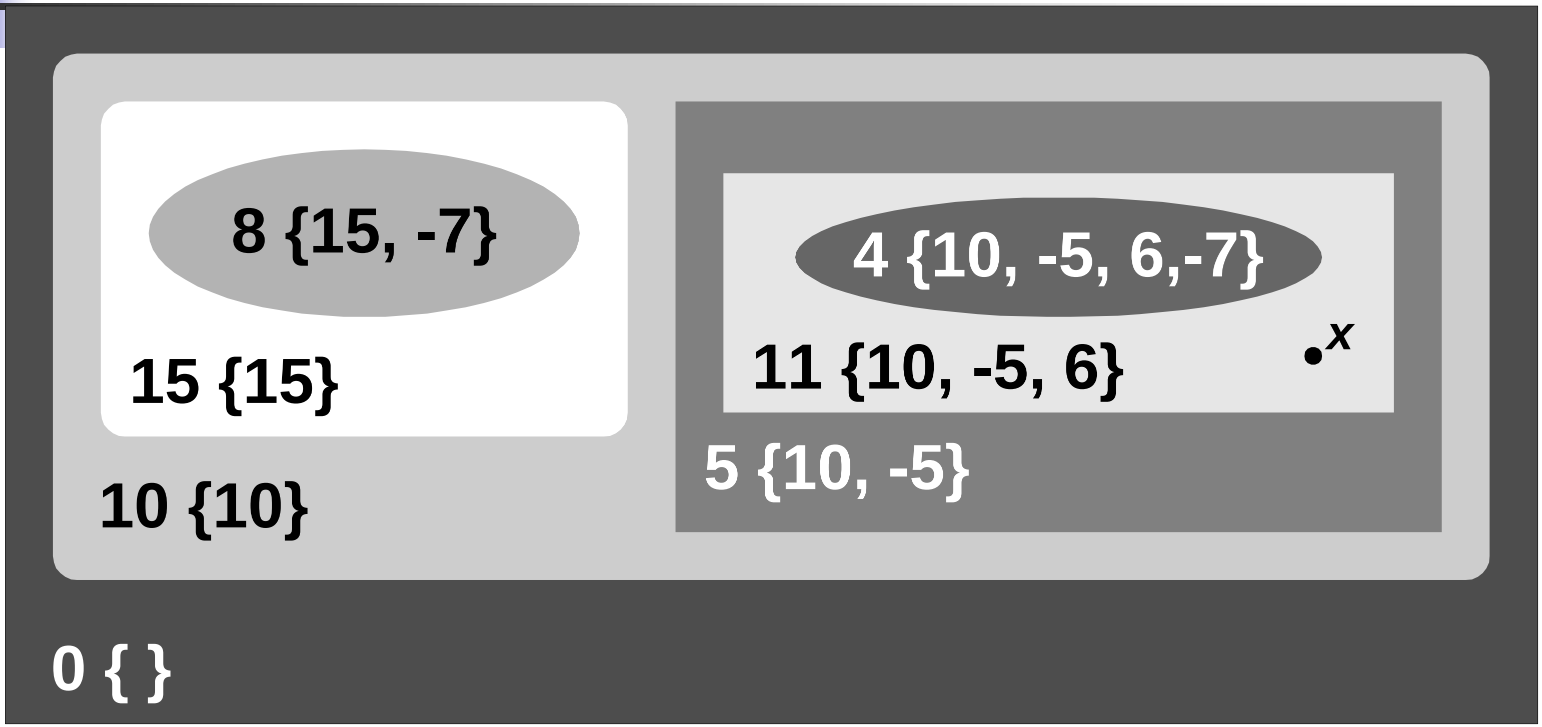
- A poset *S* is an *inf-semilattice* if every subset *K*, has an infimum $\wedge K$ in S.
- A poset *S* is an *sup-semilattice* if every subset *K*, has a supremum $\vee K$ in S.

Morphological operators in inf-semilattices





Alternating Sequences (R. Keshet, 2004)



- Alternating Sequence (AS) {10, -5, 6}
- Topographic distance |10|+|-5|+|6|=21
- Boundary-Topographic-Variation (BTV) Transform x → {10, -5, 6}
- Inverse BTV Transform 10-5+6=11

AS Semilattice (R. Keshet, 2004)

	Relation	Infimum	Supremum
1	$\{7,-2,5\} \sqsubseteq \{7,-2,6,-1\}$	$\{7,-2,5\}$	$\{7,-2,6,-1\}$
2	$\{-1,7,-9\} \not\sqsubseteq \{-5,4,-7,1\}$	$\{-1\}$	$\nexists$

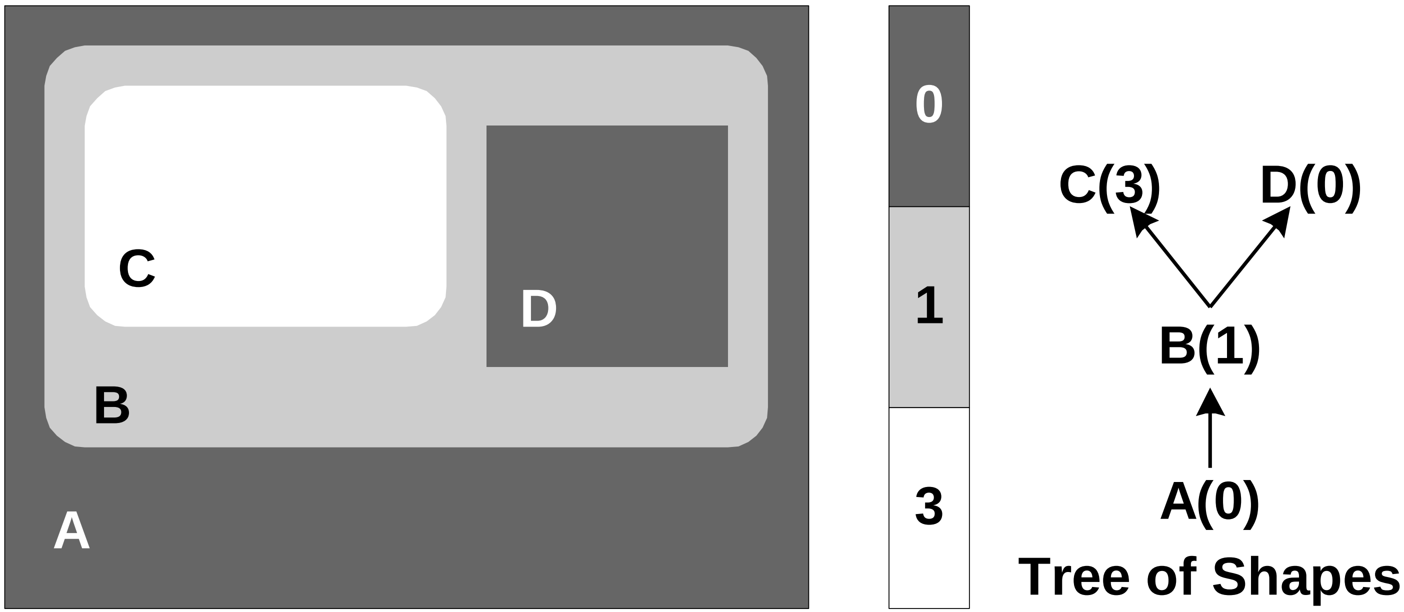
- Let V_1 and V_2 be two alternating sequences with lengths L_1 and L_2, respectively.

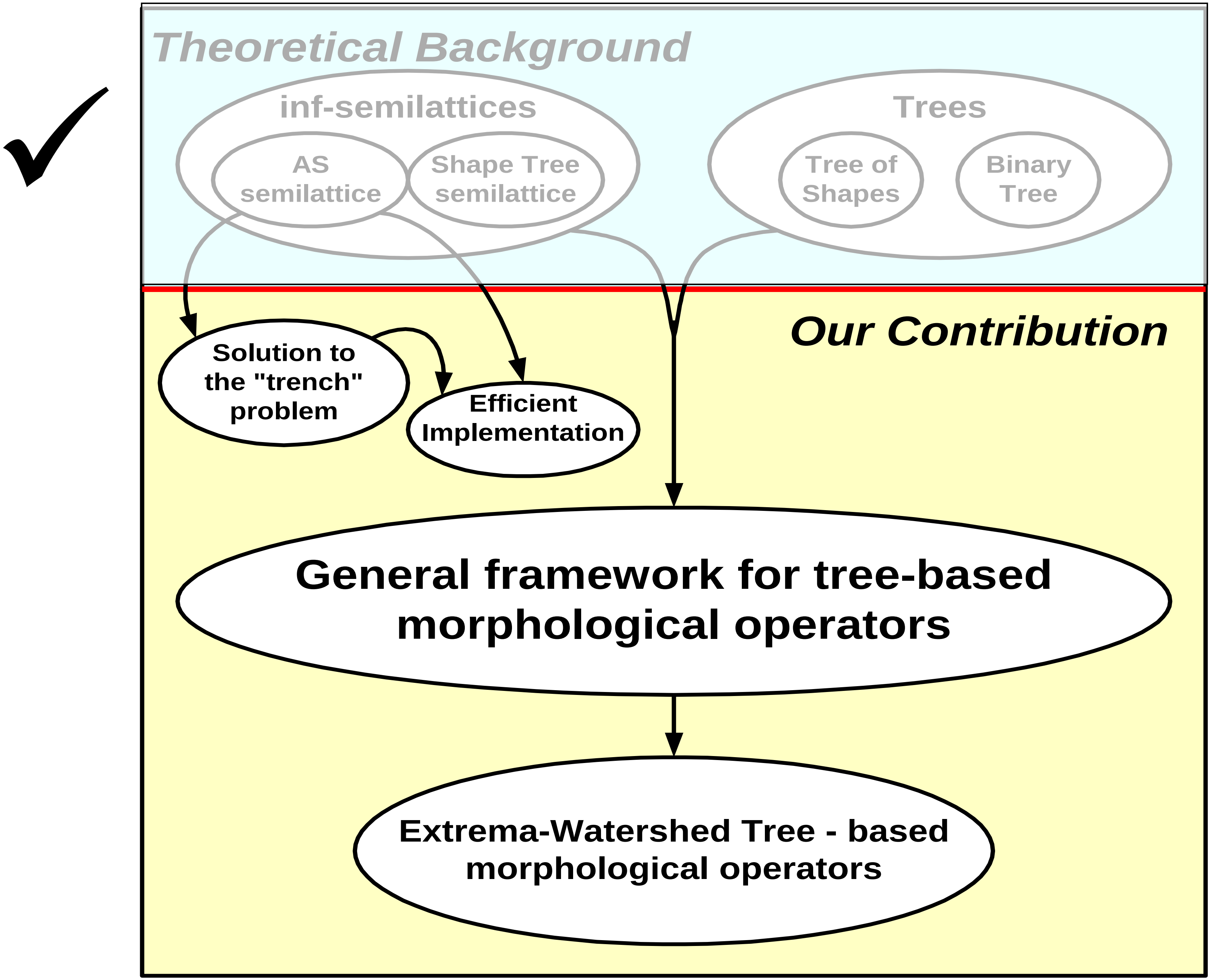
$$V_1 \sqsubseteq V_2 \iff \begin{cases} (V_1)_i = (V_2)_i, & \forall i < L_1, \\ |(V_1)_{L_1}| \leq |(V_2)_{L_1}|. \end{cases}$$

- The infimum is the common prefix, followed by the weakest of the next elements.

Shape tree semilattice (R. Keshet, 2005)

- Based on order of binary sequences.
- Uses the datum of the tree of shapes.





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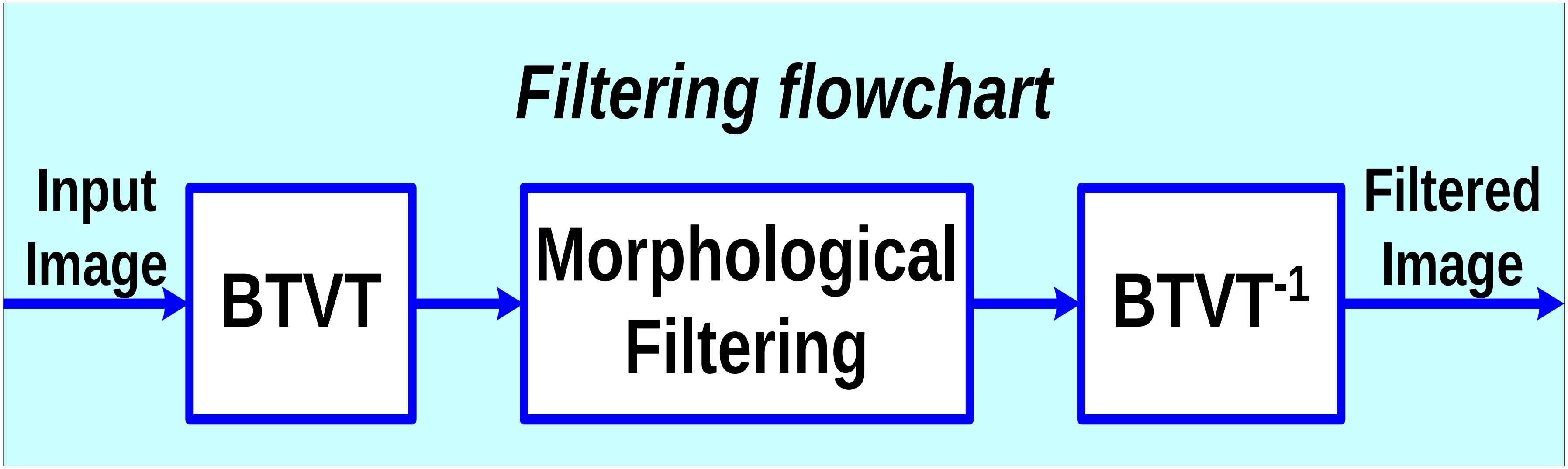
Morphological filtering in BTVT domain

- Efficient implementation of BTV transform, using intermediate tree representation of an image:
  - Working on N regions instead of pixels
  - Memory usage of O(N), instead of $O(N^2)$ for non-tree representation



Trenches problem

Original Image

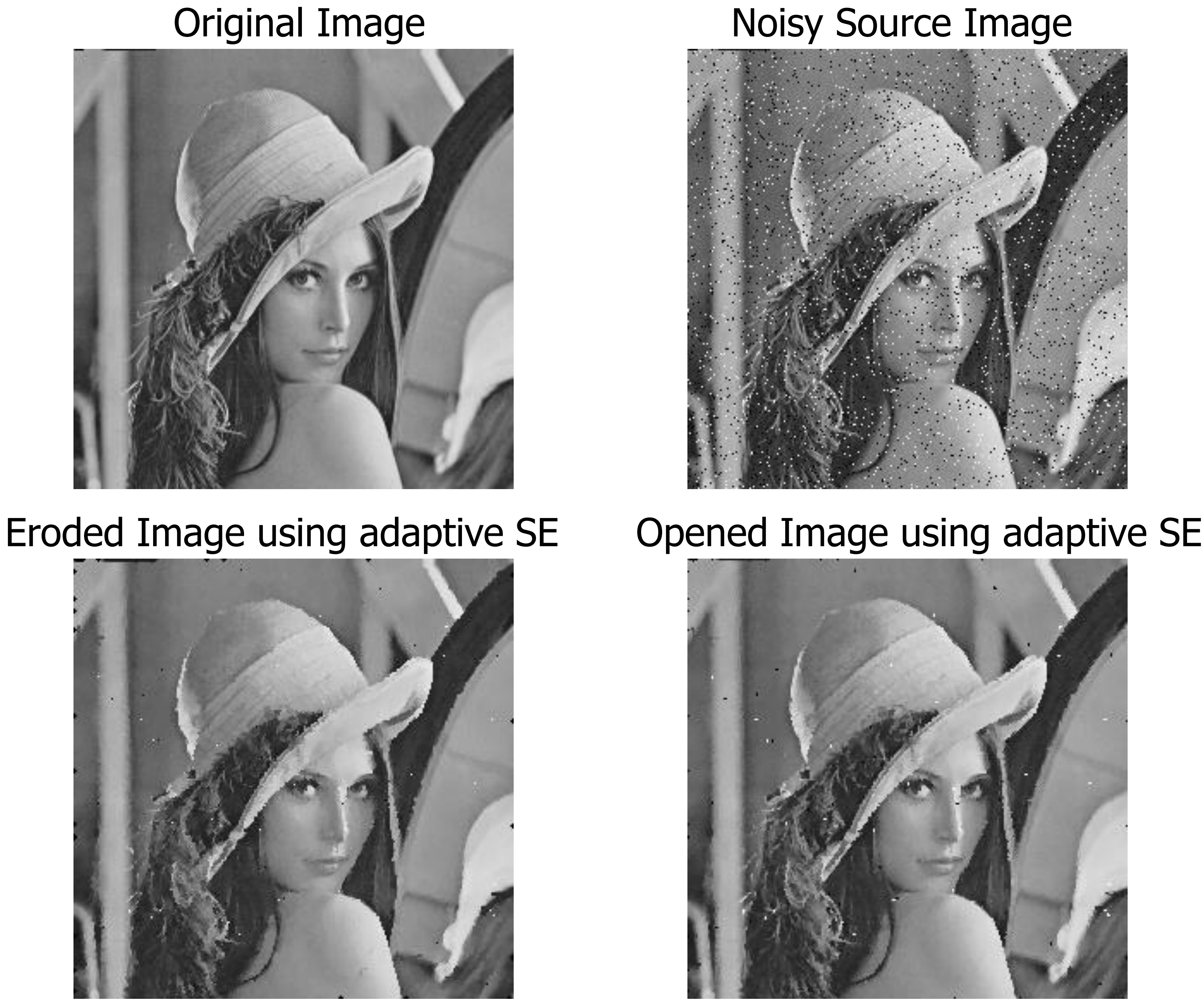
Image Eroded by cross SE

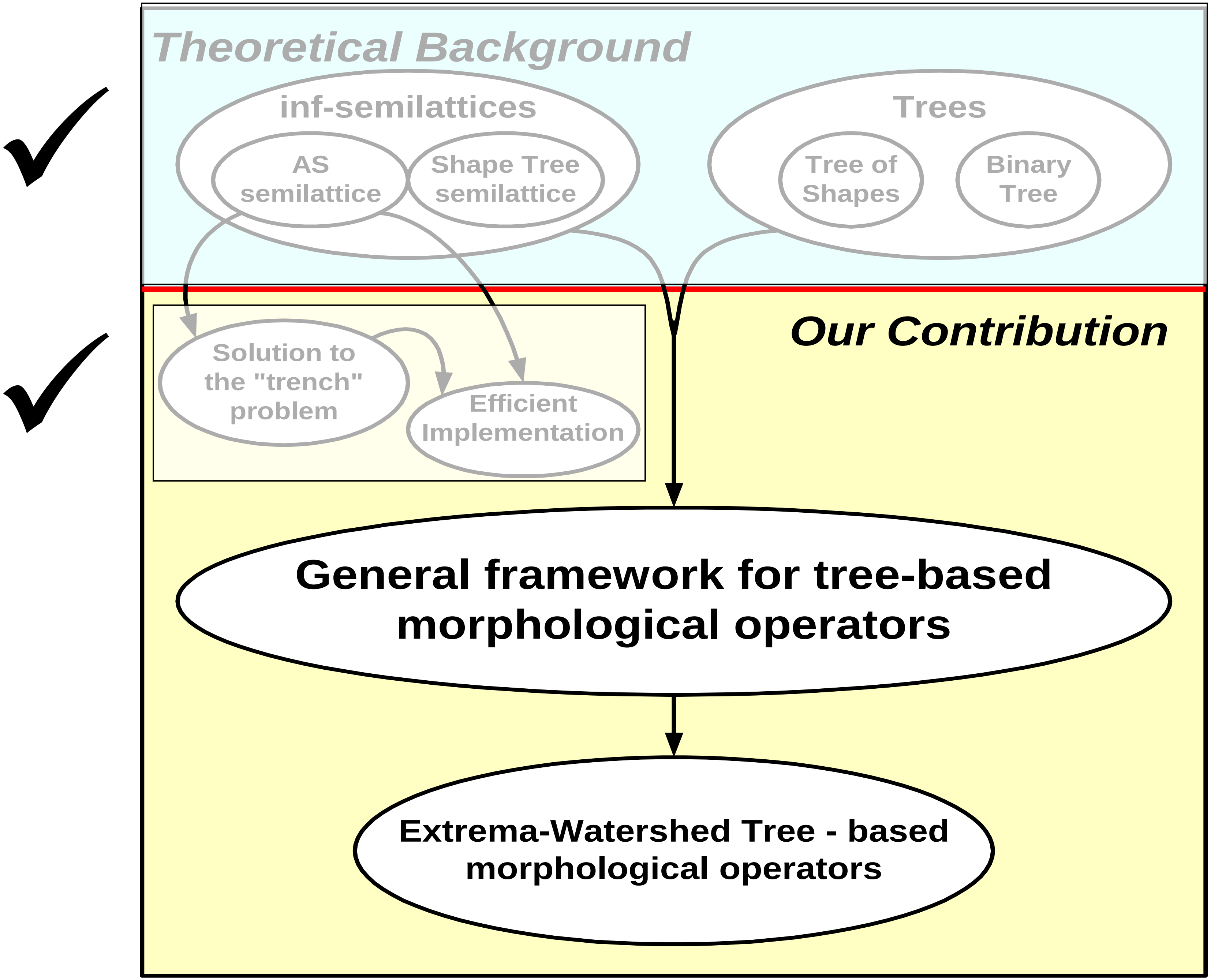
- When image is eroded, “trenches” may open on skeleton pixels
  - The BTVT is not necessarily unique at those pixels.
  - Those pixels have neighbors with very dissimilar alternating sequences.

Trenches problem - Solutions

Solution	Advantages	Disadvantages
Use adaptive SE - exclude very dissimilar neighbors	▪ Prevents trenches	▪ Limits the resulting filter performance ▪ In some applications it is necessary to save this adaptive SE for every pixel
Use adaptive SE and Store multiple BTVT options	▪ Prevents trenches ▪ Better filter performance	▪ Memory and computation time consuming ▪ In some applications it is necessary to save this adaptive SE for every pixel


Original Image
Eroded Image
Eroded Image using adaptive SE
Opened Image using adaptive SE


Original Image
Noisy Source Image
Eroded Image using adaptive SE
Opened Image using adaptive SE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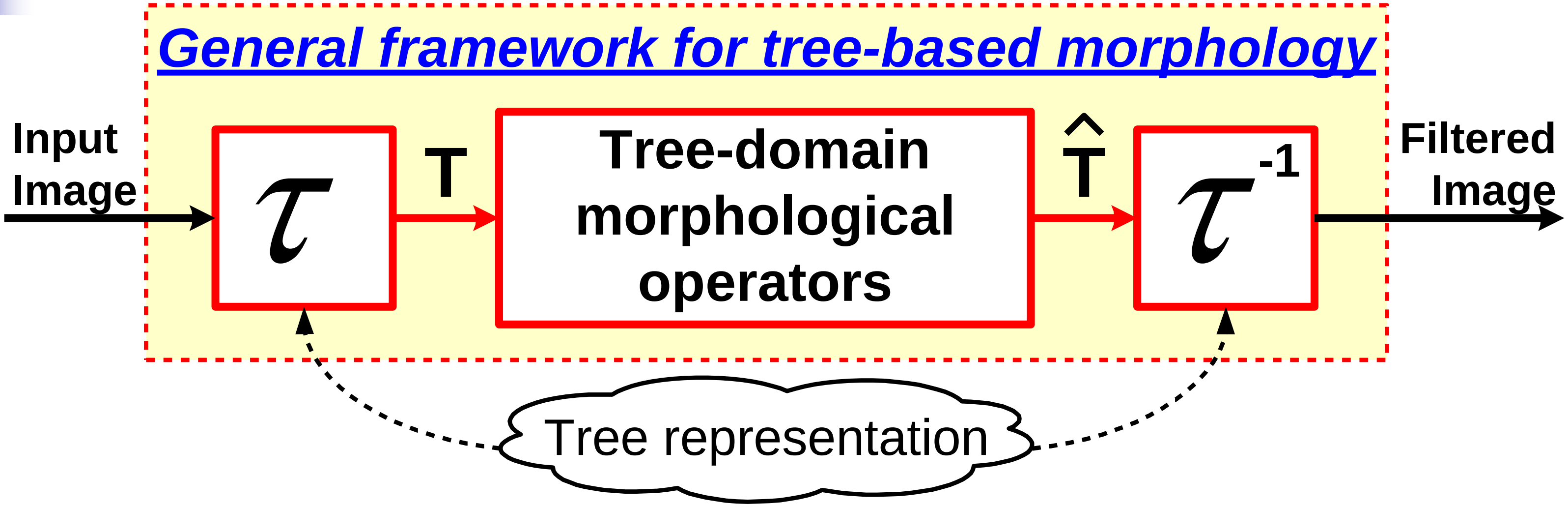
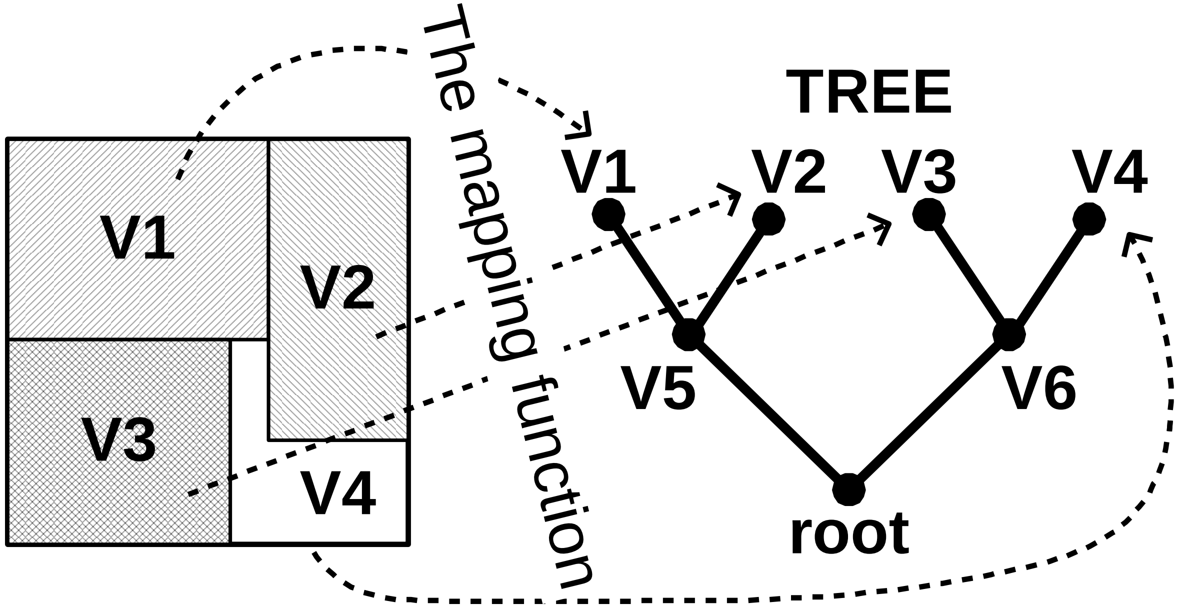
Tree-based morphology



- General framework for tree-based morphological image processing:
  - Unifies existing methods
  - Provides a solid foundation for generation of morphological operators from various tree representations.
- If the tree representation is self-dual, the resulting set of operators will also be self-dual (like Tree of Shapes).
- The heart of the proposed approach is a tree semilattice.

Tree representation

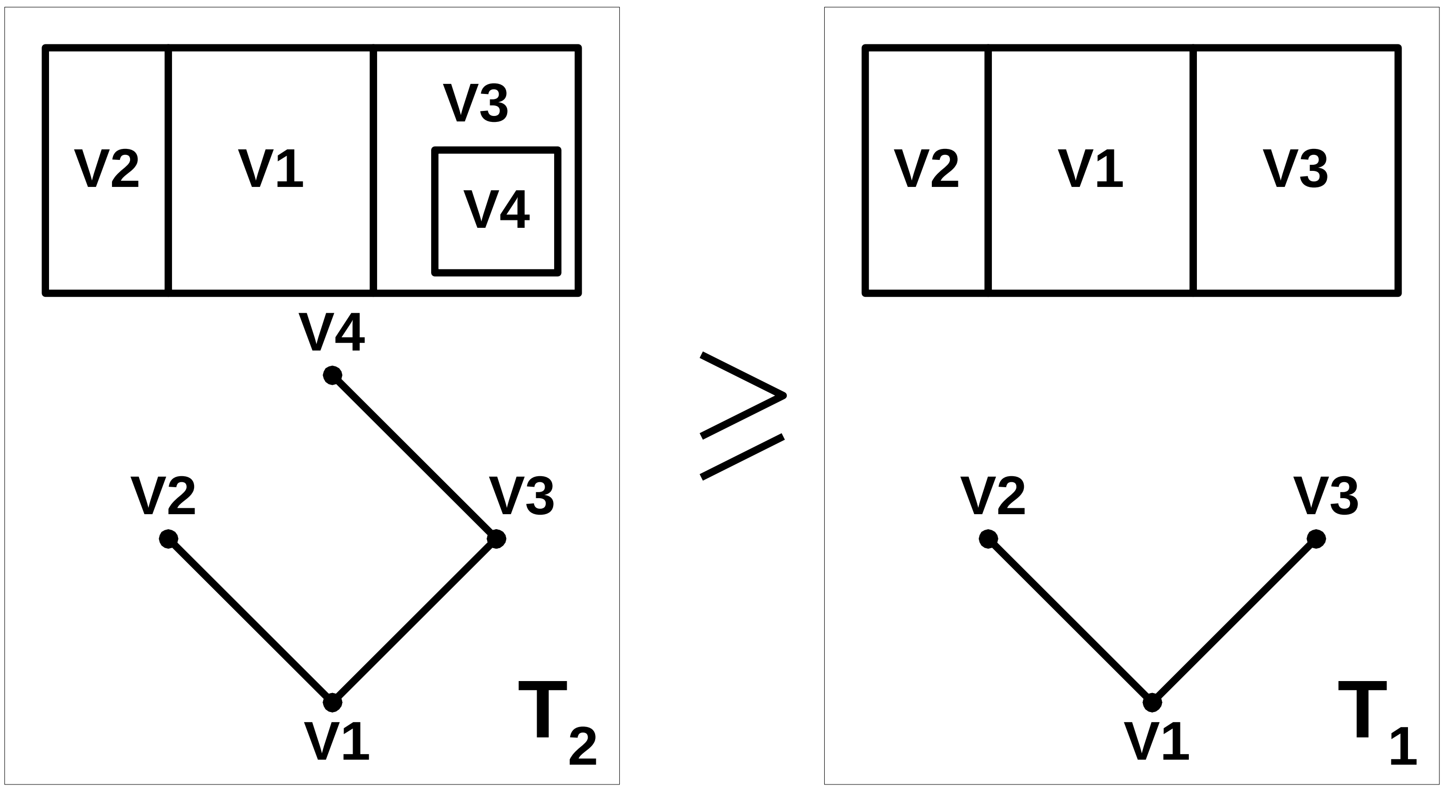
- Tree : $t = (V, E)$
- Mapping function : $M : \mathbf{R}^2 \rightarrow V$
- Tree representation : $T = (t, M)$



The tree representation order

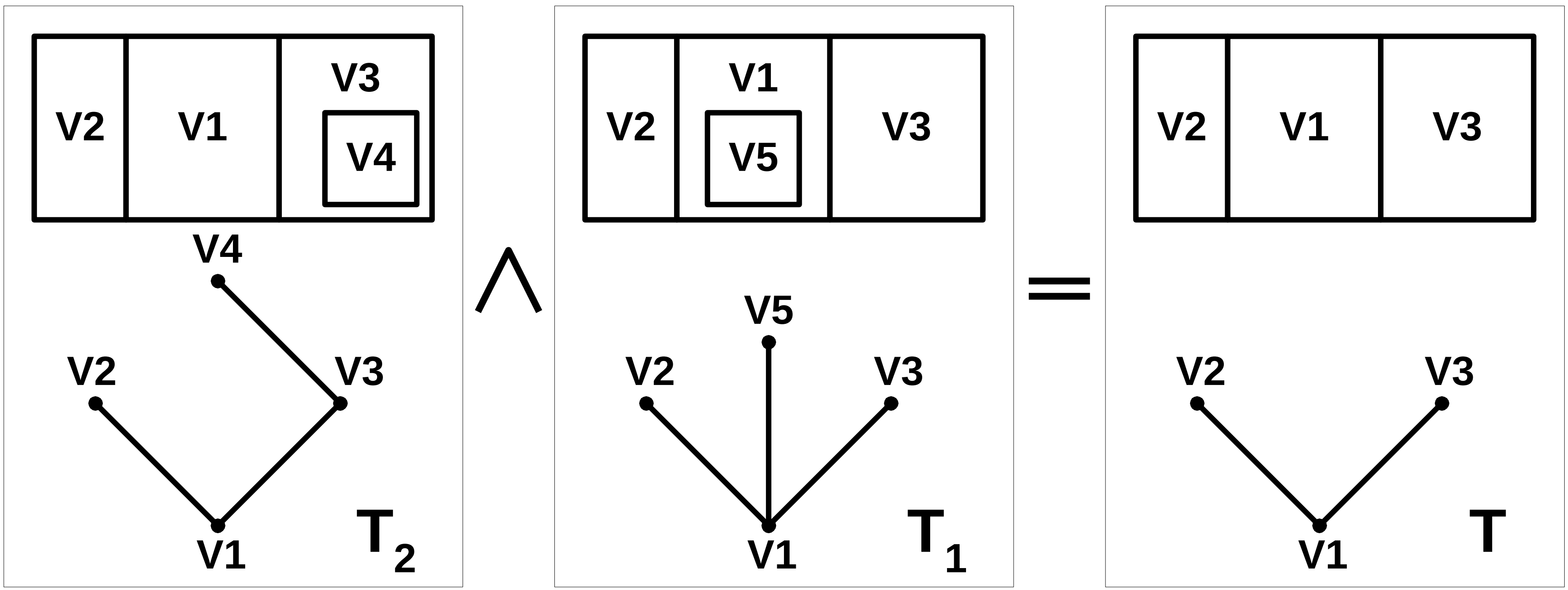
- For all $T_1=(t_1, M_1)$ and $T_2=(t_2, M_2)$

$$T_1 \leq T_2 \iff t_1 \subseteq t_2 \text{ and } M_1 \preceq_{t_2} M_2$$

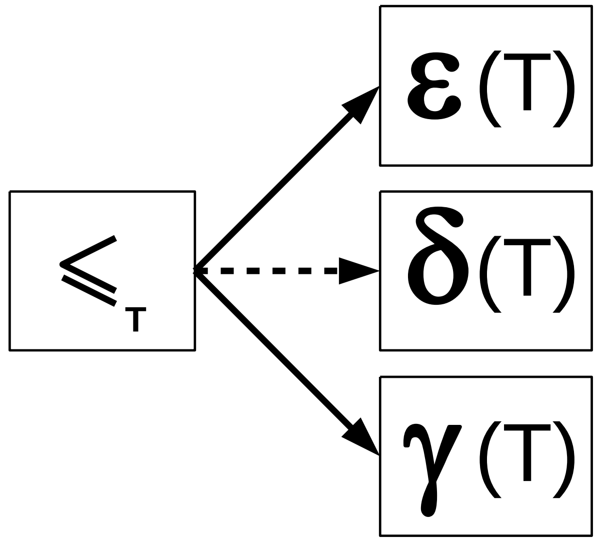


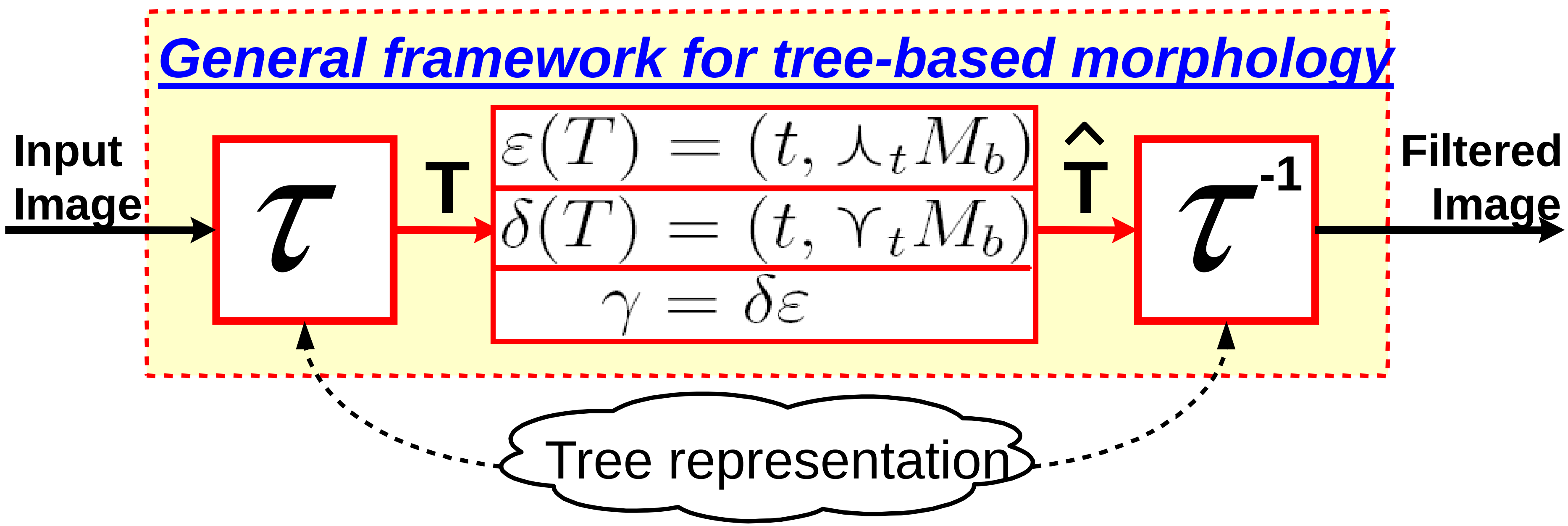
The tree representation infimum

- The tree representation infimum is given by $T=(t, M)$
  - t is the infimum of the trees t_1 and t_2,
  - M is the infimum of the mapping functions M_1 and M_2.



Tree-domain morphological operators

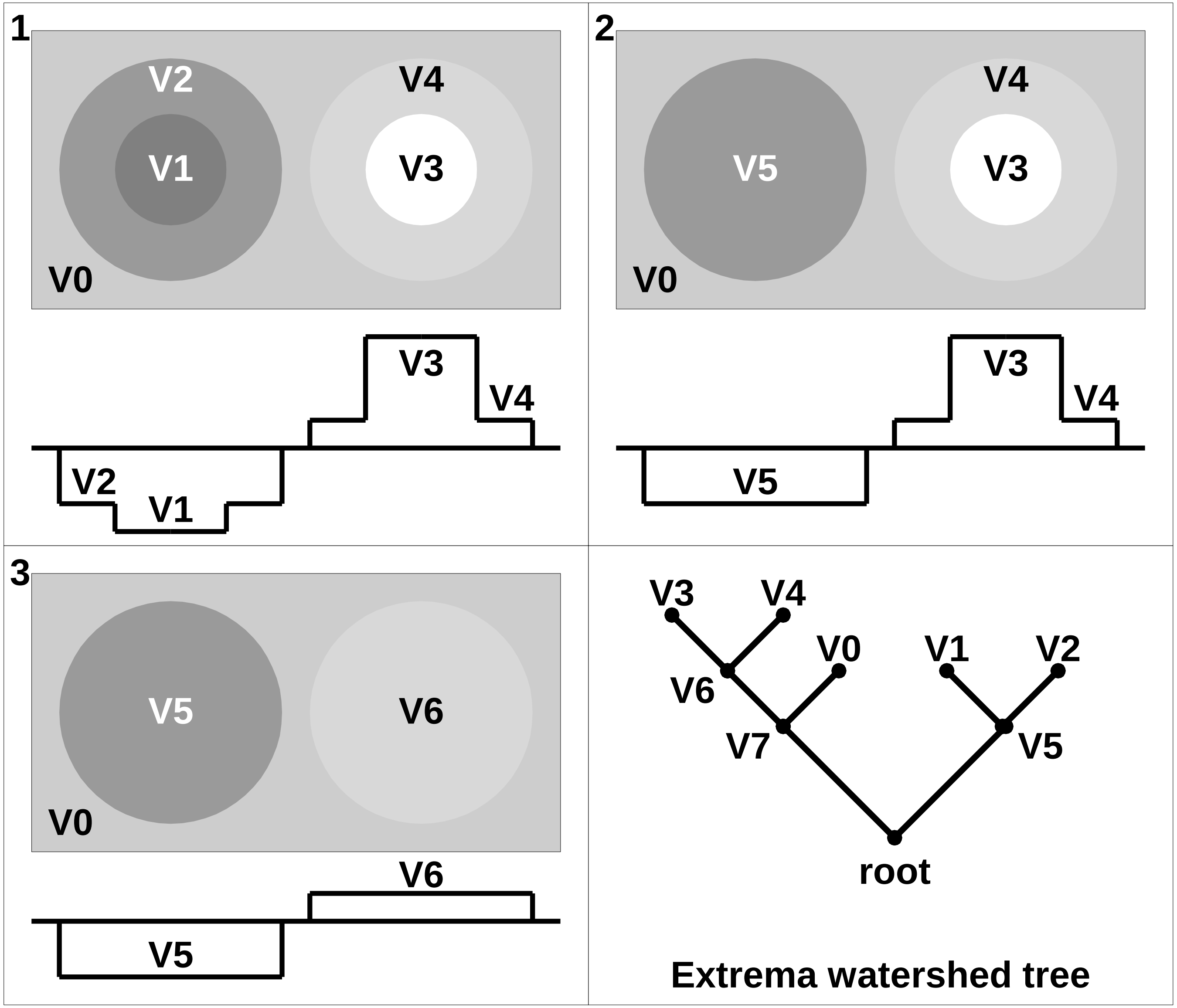




Extrema-Watershed Tree

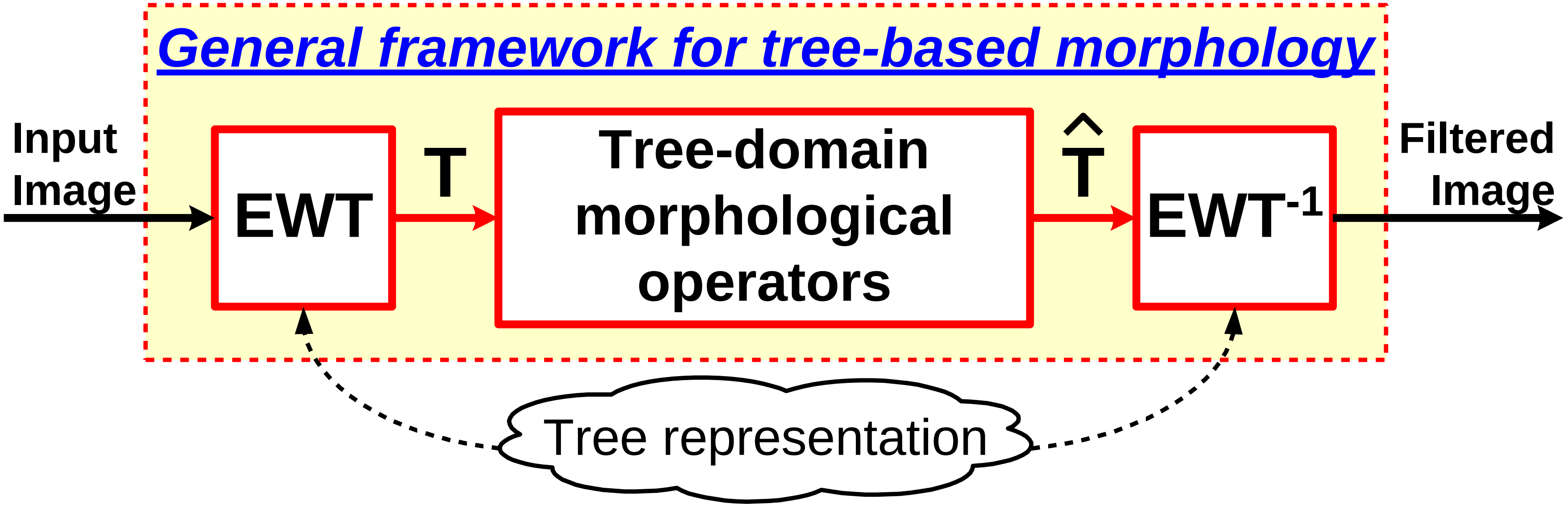
- Example of new method, obtained from the general framework
  - Stresses the strength of the general framework, as a tool for generating new, useful sets of operators.
  - Based on new self-dual tree-representation, called Extrema-Watershed Tree (EWT).
  - The new operators inherit properties of the EWT.
  - The tree is built by merging the flat zones.
  - Smallest extrema (dark or bright) regions are merged in every step.

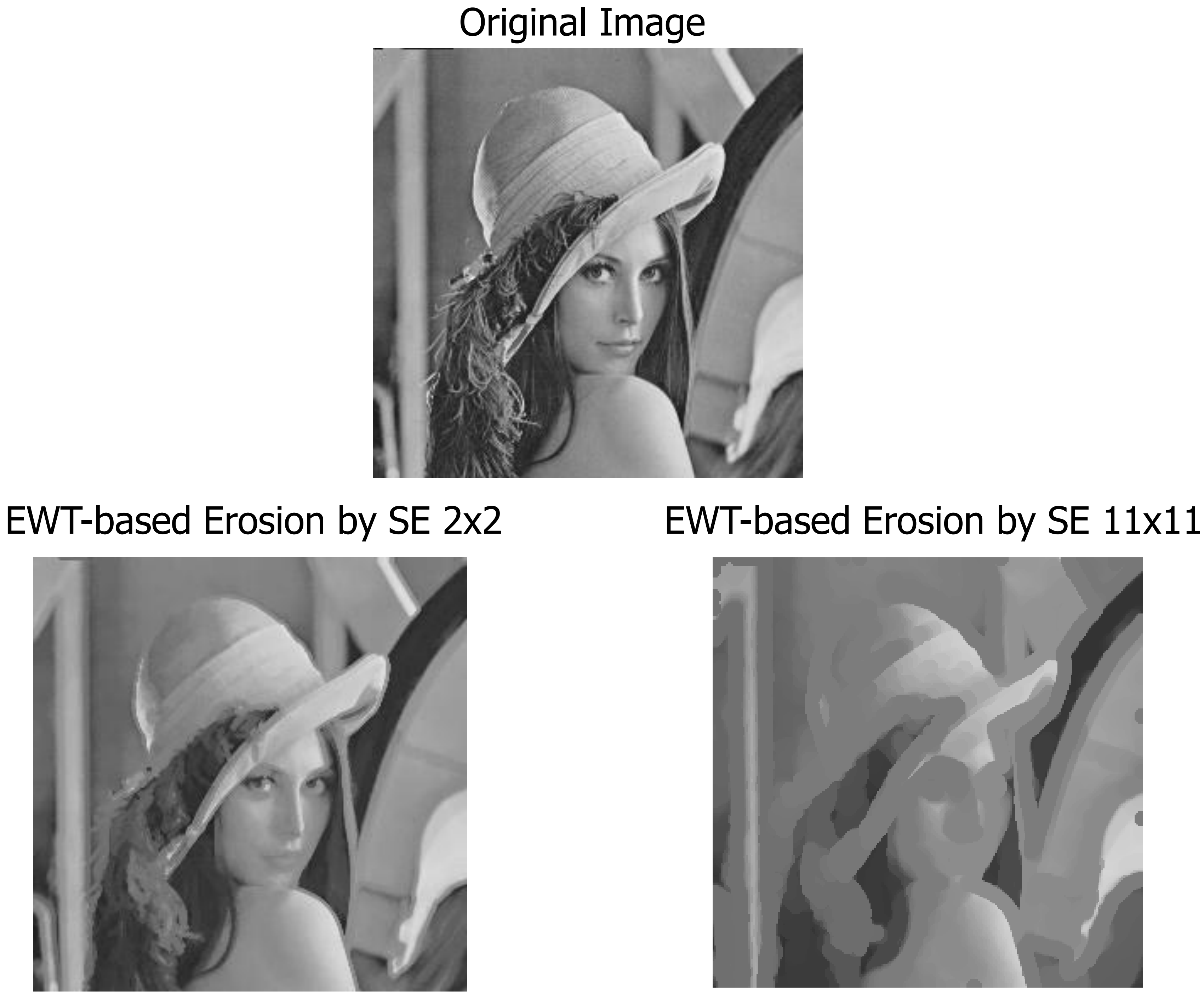
Building Extrema-Watershed Tree



EWT-based morphology

- EWT erosion and opening were obtained from the general framework.



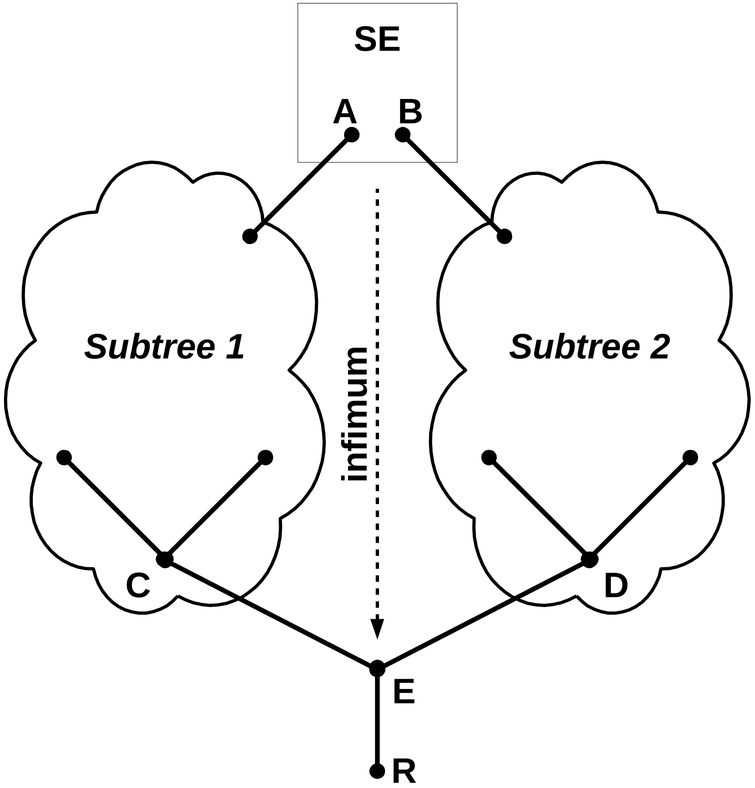

Original Image
EWT-based Erosion by SE 2x2
EWT-based Erosion by SE 11x11

Properties of EWT

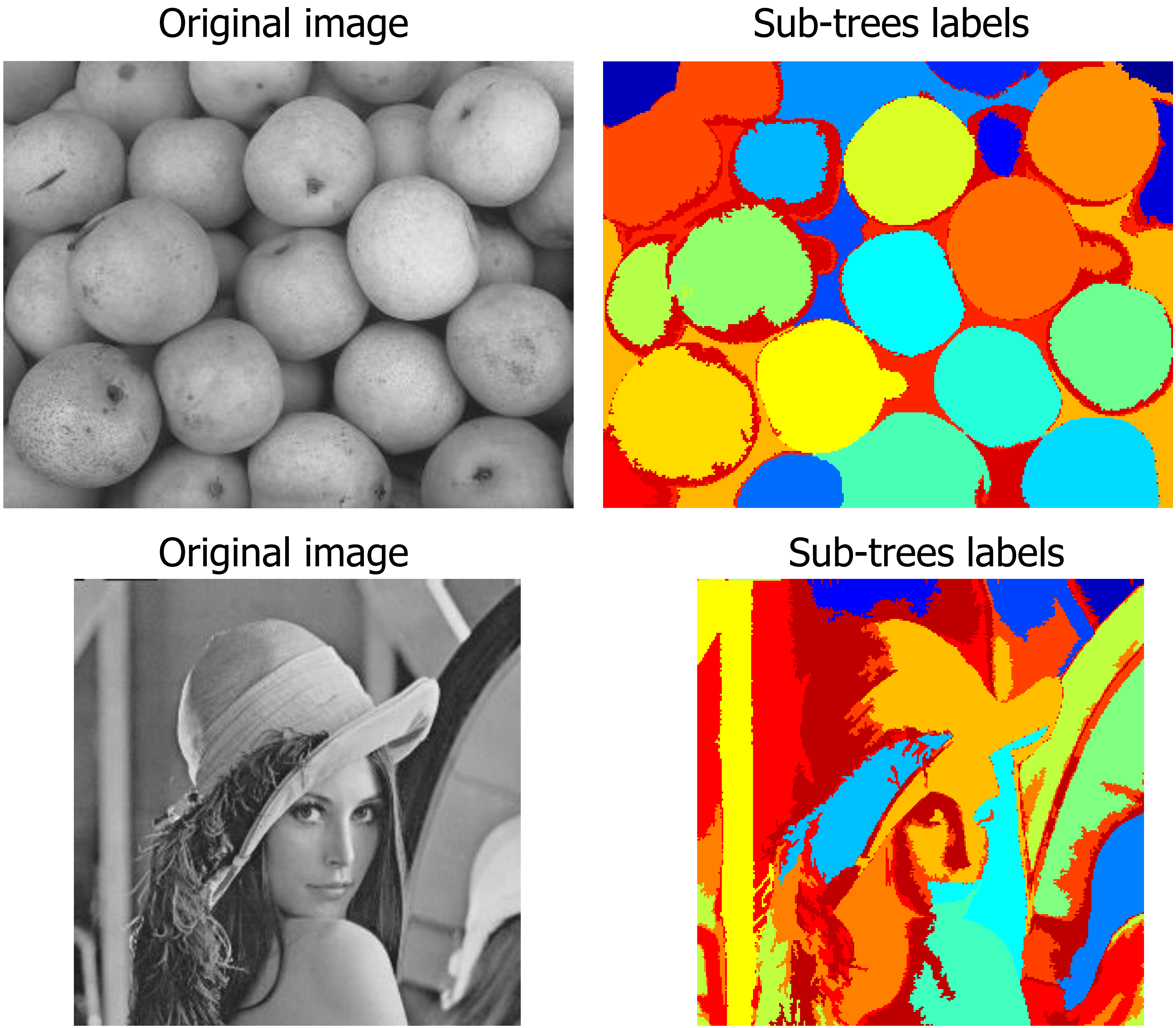
- Self-dual
- Implicit hierarchical segmentation due to:
  - Tree is created in watershed-like process
  - Small area extrema are leaves
  - Bigger flat zones are close to root
  - Vertices connected in the tree usually have similar gray levels

Implicit segmentation

- Based on the trench phenomenon
- Inherited by the "Watershed tree"



- The trench
  - is created on the border of two very "different" zones
    - "difference" is defined by the distance to the common father
  - opens a gap with gray level of the common father
- The sub-trees (common father sons) are image segments


Original image
Sub-trees labels
Original image
Sub-trees labels

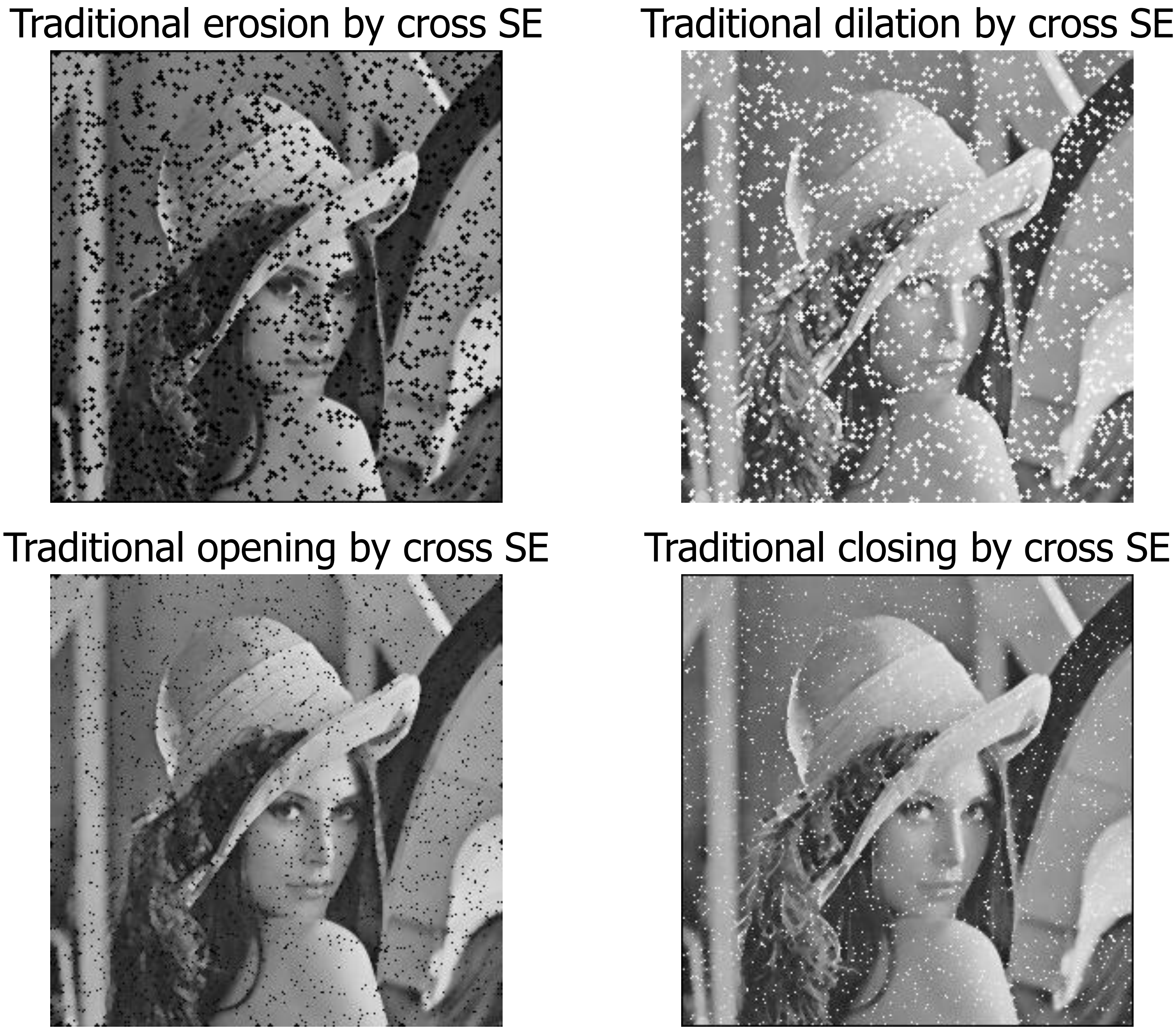
Applications

- Self-dual morphological filtering.
  - Non-connected de-noising filtering.
  - Opening by reconstruction.
- Potential for segmentation.

Original Image

Noisy Source Image

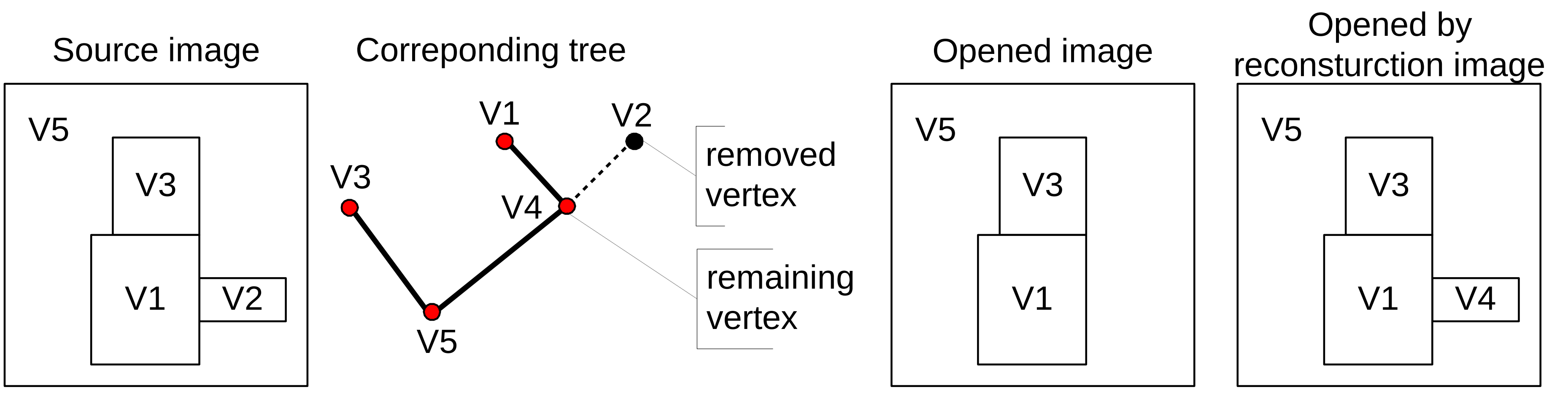
Opening of Noisy Image , based on EWT


Traditional erosion by cross SE
Traditional dilation by cross SE
Traditional opening by cross SE
Traditional closing by cross SE


Traditional opening-closing
Traditional closing-opening
Traditional median
Noisy Image Opening, based on EWT

Opening by reconstruction

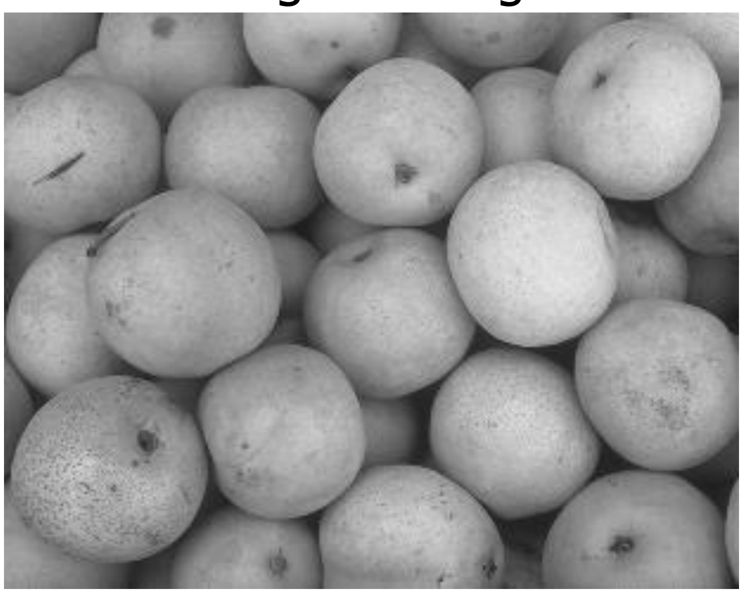
- Equivalent to tree pruning.
- A pruning criteria is whether a vertex in question does not exists in an opened image.



Noisy Source Image

Opened by reconstruction Image (EWT)

Original Image



Opened by reconstruction Image (EWT)

Conclusions

- General framework:
  - Unifies existing methods
  - Producing useful morphological operators.
- This framework was shown to be useful by:
  - Presenting the example of tree representation – EWT,
  - Giving some applications examples.
- Study of trenches:
  - Number of possible solutions in BTVT domain,
  - Potential of using trenches of EWT for segmentation

Further research topics

- Finding necessary conditions for tree representation
  - To assure existence of images semilattice, induced by trees semilattice.
- More applications based on general framework.
  - Developing more useful tree representations
  - Further exploration of segmentation capability using trenches
- Real time implementation of the proposed algorithms.