Fabian

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IDENTIFICATION OF A CLASS OF BLUR FUNCTIONS FROM BLURRED AND NOISY IMAGES

R. Fabian[1] and D. Malah
Department of Electrical Engineering
Technion — Israel Institute of Technology
Haifa 32000, Israel

ABSTRACT

In this work a robust method for the identification of a class of point spread functions (PSF) from blurred and noisy images is presented. It is assumed that the original image is passed through a linear two-dimensional blurring system and that wide-band noise is added to the observed image. Two types of blurring are considered: motion blur and out-of-focus blur.

The fact that the spectra of these blurring functions have periodic zeros, is the basis of an already known blur identification method. This periodicity is manifested by distinct negative point or circle impulses in the cepstral domain. The location of these impulses allows the identification of the parameters of the blur functions of the type considered. However, this method is found to be highly sensitive to noise. We propose therefore the following improvements to the above basic method: First, adding a preprocessing stage for noise reduction, using a modified spectral subtraction approach, and second, applying an adaptive comb-like window (lifter) in the cepstral domain to enhance the blur parameter identification.

The proposed algorithm is found to provide adequate identification of blur function parameters from noisy blurred images with signal-to-noise ratio down to 0dB for motion blur and 3dB for out-of-focus blur, as compared to 20 dB for the original method.

1. INTRODUCTION

The problem of restoring noisy images has been a difficult challenge for many years and is addressed in numerous publications. The main trends and methods in restoration can be found in any already "classic" book [1].

The major part of image restoration algorithms require some knowledge about the degradation process and associated parameters. These include the classical Wiener filter restoration techniques [1], the more recent iterative restoration algorithms [15,16], and others [9,17].

The problem is that the information required is not always available and that the restoration results were found to be highly dependent on the blurring system model used, and on the accuracy by which its parameters are identified from the degraded image. The approaches taken in many image restoration works can be put into one of the following two categories:

a. Identification of the PSF parameters in order to use it later in one of the known restoration algorithms.

b. Incorporation of the identification procedure in the restoration algorithm.

The work of Gennery [2] who tried to identify the PSF parameters in the spectral domain, the work of Mitre and Fleuret [7], for identification both in the space and spectral domains, and the works by Cole [6] and Cannon [3,4] for identification in the log-spectral and cepstral domains, all fall in the first category. The works of Biemond and Putten [12] and Tekalp and Kaufman [10,11] can be considered as members of the second category. The principal attitude in [10,11,12] is to assume that the degraded image can be modeled as a product of an ARMA system where the autoregressive part represents the image, and the moving average part represents the degradation process.

Cooke and Dorrani [8] assume a new model for the degraded imaged formation and propose an iterative process for combined identification and restoration, thus also belonging to the second category.

The main drawback of all the above mentioned methods is the high sensitivity to additive noise. The relatively simple methods of [2] and [7] are restricted to very high signal-to-noise ratios (SNR) of the degraded image under test. In papers [6,8,10,12], the lowest SNR reported is 20dB. This was also the result in our examination of Cannon's method [4].

In this paper we present a robust method for identifying the PSF parameters from motion and out-of-focus blurred images with additive noise, which is partly based on the approach in [4].

2. BLURRING-SYSTEM MODEL

We assume that the image acquisition process is as depicted in Fig. 1 with a linear space-invariant imaging system.

Mathematically, it can be described as follows:

$$g(x,y) = f(x,y) * h(x,y) + n(x,y) \tag{1}$$

where: $f(x,y)$ - original image; $g(x,y)$ - observed image; $h(x,y)$ - point-spread-function of the imaging system; $n(x,y)$ - additive noise; $*$ - convolution operator.

In the Fourier transform domain, equation (1) takes the form:

$$G(u,v) = F(u,v) \cdot H(u,v) + N(u,v) \tag{2}$$

2.1 Blur Functions

As noted above, two types of blur functions are dealt with in this paper: motion blur, caused by relative motion between the object and the camera along the x axis during exposure time, and out-of-focus blur, caused by mislocation of the camera's lens having a circular aperture.

For motion blur, $h(x,y)$ is given by a one dimensional rectangle:

$$h(x,y) = \begin{cases} \frac{1}{d} & -\frac{d}{2} \le x \le \frac{d}{2} \; ; \; y=0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where d is the "blur-length" and is proportional to the relative velocity between the camera and the object and to the film exposure time. The Fourier transform of $h(x,y)$ in this case is:

$$H(u,v) = \frac{\sin(\pi du)}{\pi du} = sinc\ (\pi du) \tag{4}$$

The amplitude of $H(u,v)$ is characterized by periodic zeros on the u axis, which occur at

$$u = \pm\frac{1}{d}, \pm\frac{2}{d}, \pm\frac{3}{d}, \cdots \tag{5}$$

For out-of-focus blur, $h(x,y)$ is assumed here to be given by the cylinder:

$$h(x,y) = \begin{cases} 1/\pi R^2 & x^2+y^2 \le R^2 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where R is the "blur radius" and is proportional to the extent of defocussing. The Fourier transform of $h(x,y)$ in this case is:

$$H(u,v) = J_1(\pi Rr)/\pi Rr \tag{7}$$

where $r^2 = u^2 + v^2$ and $J_1(\cdot)$ is the first order Bessel function. The

[1]Presently with RAFAEL – Armament Development Authority, Haifa, Israel

amplitude of $H(u,v)$ is characterized by "almost-periodic" circles of radius r, at which $H(u,v)$ takes the value zero. This occurs at values of r satisfying:

$$2\pi Rr = 3.83,\ 7.02,\ 10.2\ 13.3,\ 16.5, \cdots \qquad (8)$$

It is seen therefore that these two types of blurs are each characterized by a PSF which requires only a single parameter for its complete determination.

2.2 Cepstral Representation

The cepstral representation of the blur function $h(x,y)$ is given by:

$$C_h(p,q) = F^{-1}\{\log|H(u,v)|\} \qquad (9)$$

where F^{-1} denotes the inverse Fourier transform. Since $|H(u,v)|$ is real and even, so is $C_h(p,q)$.

For motion blur, it is recalled that the distance between adjacent zeros of the blur function spectrum is $\frac{1}{d}$. Representing this function in the cepstral domain results in a distinct negative pulse at a distance d from the origin and replicas of this pulse at integer multiples of d.

For our-of-focus blur $H(u,v)$ has "almost-periodic" circles of zero value where the distances between adjacent circles is approximately $1/(2R)$. Representation of the blur function in the cepstral domain gives a distinct circle of negative amplitude with a radius of approximately 2R and replicas of this circle with radii which are approximately multiples of 2R.

3. CANNON'S APPROACH

In his works [3,4], Cannon found out that the noise added to the blurred image is the main cause for the "disappearance" of zeros of the PSF in the spectral domain and therefore prevents their identification in the cepstral domain. He proposed therefore the following scheme:
The assumption that the image and the noise are samples of a stationary random process gives, from (2):

$$P_g(u,v) = P_f(u,v)\cdot|H(u,v)|^2 + P_n(u,v) \qquad (10)$$

where $P_g(u,v)$, $P_f(u,v)$, and $P_n(u,v)$ denote the power spectra of $g(x,y)$, $f(x,y)$, and $n(x,y)$, respectively.

An estimate of $P_g(u,v)$ can be obtained using Welch's algorithm [14]. This is done by dividing the image into sub-images, multiplying each sub-image by a 2-D Hamming window, computing its squared magnitude Fourier transform (modified periodogram), and averaging over all sub-images, resulting in

$$\hat{P}_g(u,v) = \overline{P_f(u,v)}\cdot|H(u,v)|^2 + \overline{P_n(u,v)} \qquad (11)$$

where the overbar indicates the effect of the above process on the original power spectra.

Assuming that the noise is white, $\overline{P_n(u,v)}$ converges (as the number of sub-images grows) to a constant which equals to the noise variance. It is evident that if the noise variance is sufficiently small, conspicuous negative peaks in the cepstral domain can be expected at locations which are multiples of the blur parameters, as explained above, and they will dominate the form of $C_{\hat{g}}(p,q)$- the cepstral representation of $P_g(u,v)$. Identification can thus be achieved by a careful examination of the one-dimensional sequence $C_{\hat{g}}(p,0)$ in the cepstral domain.

As was mentioned earlier, the main drawback of this algorithm is its high sensitivity to additive noise. The main goal of this paper is therefore to modify the above approach and make it more robust in presence of high level noise. The first step in our proposed algorithm is noise reduction by spectral subtraction - as elaborated next.

4. NOISE REDUCTION BY SPECTRAL SUBTRACTION

A detailed development and description of the spectral subtraction algorithm is given in [13]. Here we briefly present the main result. Let $b(x,y)$ denote the blurred only (noise free) image. Hence equation (1) takes the form:

$$g(x,y) = b(x,y) + n(x,y) \qquad (12)$$

Estimation of the Fourier transform of $b(x,y)$, $B(u,v)$, from $G(u,v)$ by using the power spectrum method [1], results in the following formulation of the spectral subtraction algorithm [13]:

$$\left.\begin{array}{ll} |\hat{B}(u,v)| = [\,|G(u,v)|^2 - \alpha\ P_n(u,v)]^{1/2} & \text{if } |G(u,v)|^2 \geq \alpha\ P_n(u,v) \\ |\hat{B}(u,v)| = \varepsilon & \text{otherwise} \\ \measuredangle \hat{B}(u,v) = \measuredangle G(u,v) & \end{array}\right\} \qquad (13)$$

where α is a coefficient used to control the subtraction extent and ε is a small constant used to avoid numerical difficulties when taking the logarithm of (13).

The method of estimating $P_n(u,v)$ is explained in Section 5. Note that in our application no use is made of the phase, as we need not reconstruct here the enhanced image.

Because of image nonstationarity, it is common to apply the spectral subtraction technique to sub-images of the given image and then recombine the enhanced sub-images to obtain the restored image. It appears therefore that the integration of this enhancement technique with the spectral averaging of sub-images used in Cannon's algorithm can be done quite efficiently as depicted in Fig. 2. However, the performance of this approach for blur parameter identification, as well as some other integration approaches we examined, was found to be lower than using the spectral subtraction approach on the whole image, as described in Section 5. It should be emphasized that since the goal here is *not* the restoration of the image but the extraction of the blur function parameter, our judgement of the noise reduction technique applied, is based only on the ability to identify the blur parameters and not on the enhancement of the image.

5. ALGORITHM DESCRIPTION

The proposed algorithm has two stages. In the first stage a form of the spectral subtraction method is employed for noise reduction. In the second stage the enhanced spectral magnitude function is transformed to the cepstral domain and the identification procedure is completed using an adaptive "comb-like" window (lifter).

5.1 Stage I: Noise Reduction

The noise reduction procedure in this stage follows the spectral subtraction formulation given in (13) with $\alpha = 1$ (the value of α was carefully selected to enhance performance), but is applied to the whole image, without dividing it into sub-images.

It remains to get a good estimate of $P_n(u,v)$. This is done here by using a "median-complement" image, defined below, to get an estimate of the noise, $\hat{n}(x,y)$, from which $P_n(u,v)$ is computed. A median filter [1] is defined as follows:
Let $\{a_n\}_{n=1}^{2K+1}$, be an input sequence of length $2K+1$. $\{a_n\}$ is reorded to get the sequence $\{b_n\}_{n=1}^{2K+1}$, according to the law,

$$b_{n_2} \geq b_{n_1} \leftrightarrow n_2 > n_1 \qquad (14)$$

then:

$$\text{median}\ \{a_n\}_{n=1}^{2K+1} = b_{K+1} \qquad (15)$$

I.e., the median is given by the center element of the rank ordered sequence $\{b_n\}$.
A "median-complement" filter is thus defined by:

$$\text{median–complement}\ \{a_n\}_1^{2K+1} = a_{K+1} - b_{K+1} \qquad (16)$$

Applying this procedure to the problem at hand results in:

$$\hat{n}(x,y) = \text{median–complement}\ \{g(x,y)\} \qquad (17)$$

and we let

$$\hat{P}_n(u,v) = P_{\hat{n}}(u,v) = |\hat{N}(u,v)|^2 \qquad (18)$$

where $\hat{N}(u, v)$ is the Fourier transform of $\hat{n}(x, y)$. The filter used was a one-dimensional median filter of length 3, i.e., $K=1$.

The block diagram of this stage appears in Fig. 3a. Note that a 2-D Hamming window is used prior to the Fourier transformation.

5.2 Stage II: Blur Parameter Identification

We have seen already that the cepstral representation of the PSF is dominating the form of $C_{\hat{b}}(p, q)$ — the cepstral representation of the blurred image. In the motion-blur case, a distinct negative pulse appears on the p axis. In the out-of-focus blur case, a distinct circle with negative values appears in this domain, crossing the p axis.

Although it seems sufficient to examine the sequence $C_{\hat{b}}(p, 0)$ and look for a negative dominant pulse, in the presence of noise this was found to be effective only for the motion-blur case. For out-of-focus blur the following approach is taken:

First, $C_{\hat{b}}(p, q)$ is converted to a polar coordinate representation to get $C_{\hat{b}}(r, \theta)$. Then, the sequence $C_{\hat{b}}(r)$ is created by averaging over θ:

$$C_{\hat{b}}(r) = \frac{1}{2\pi} \int_0^{2\pi} C_{\hat{b}}(r, \theta)\, d\theta \qquad (19)$$

Thus, all the information available along a circle of some radius r_0 in $C_{\hat{b}}(p, q)$ is concentrated at the single point $C_{\hat{b}}(r_0)$.

Since a one-dimensional sequence is actually used for identifying the blur parameter. We examined ways for reducing the amount of computations. For the motion blur case this is quite simple, since by the "projection-slice" theorem [5],

$$C_{\hat{b}}(p, 0) = F^{-1}\{\, s(u)\,\} \qquad (20)$$

where

$$s(u) = \int_{-\infty}^{\infty} \log |B(u, v)|^2\, dv \qquad (21)$$

i.e., $C_{\hat{b}}(p, 0)$ can be obtained from the inverse 1-D Fourier Transform of the 1-D sequence obtained by averaging $\log |B(u, v)|^2$ along the columns (following, of course, discretization of the (u, v) frequency axes).

For the out-of-focus case, an attempt to apply a similar theorem in polar coordinates (to obtain $C_{\hat{b}}(r)$) did not lead to the sought simplification since a Hankel transform is involved, which is not simple to compute. For convenience of the pursuing development we denote the 1-D sequence obtained in both cases $C_{\hat{b}}(m)$.

We mentioned before that the desired negative pulse in the cepstral examined sequence is accompanied by its replicas and other pulses which result from the additive noise and the image itself.

Usually these pulses are of lower amplitudes than the desired first pulse, but sometimes, because of the additive noise, this is not the case and mis-identification occurs. The following algorithm uses a new criterion in order to suppress undesired negative pulses and to enhance the desired one.

Every point in the cepstral sequence $C_{\hat{b}}(m)$ is assigned a new value according to:

$$C(m) = \begin{cases} 0 & \text{if } C_{\hat{b}}(m) \geq 0 \\ |C_{\hat{b}}(m)| / [\frac{1}{M} \sum_{l \in A} (C_{\hat{b}}(l))^2]^{1/2} & \text{if } C_{\hat{b}}(m) < 0 \end{cases} \qquad (22)$$

where:

m – the examined point index.

A – the set of all the points in $C_{\hat{b}}(\cdot)$ having a negative value, and:

- are of index greater than 3
- do not coincide with the examined point
- are not within the region of ± 1 of an integer multiple of the m.

M_m – total number of points in A.

It is noted that every point $l \in A$ can be considered as a "disturbance" when the point m is examined, since it is negative and is not a replica of m. Therefore, the denominator of $C(m)$ contains the "average disturbance" related to this point.

Since $C(m)$ is computed for all points in the cepstral sequence $C_{\hat{b}}(m)$, the algorithm actually scans this sequence with an adaptive 'comb-like' window (lifter) with pass- and stop-bands which are matched to every point in the sequence.

In Fig. 3 — a graphic description of the 'comb' window is shown.
The following three cases are of interest:

a. m is the point of the desired pulse - in this case the integer replicas of m will not take part in the "disturbance" computation. All other points in A contain only 'random' noise and $C(m)$ will get a high positive value.

b. m coincides with one of the desired-pulse replicas - in this case A will contain the desired pulse, some of its replicas and the other 'random' noise points. $C(m)$ will thus get a low value.

c. m is some point that does not fit the description in the former two cases — in this case the "disturbance" consists of the desired pulse and most of its replicas beside some other 'random' noise points. Thus $C(m)$ will get even a lower value than in case b.

The final stage for identification is a peak-detector activated on the sequence $C(m)$.

A block diagram of this stage appears in Fig. 3b.

6. RESULTS

An experiment was conducted in which pictures were synthetically degraded by a computer, where the extent of blur and noise corruption was completely controllable.

The algorithm was tested for various values of the blur parameters. For each blur type and blur parameter, noise was added in order to determine the minimum value of input SNR that still allows identification of the blur parameter.

For motion-blur, the parameter d (blur length) was given values such as 8, 11, 19, ... pixels and many others. For out-of-focus blur, the blur radius R was given various values, for example R=8,12, ... and more.

The Signal-to-noise Ratio is defined here as the ratio of the signal variance to the noise variance. Examples of the results obtained in the experiment appear in table 1:

Table 1: Minimum SNR Required for Blur Parameter Identification

Blur Type & Extent →	Motion $d=11$	Motion $d=19$	Defocus $R=8$	Defocus $R=12$
SNR [dB] →	0.7	-0.4	3.1	2.8

7. CONCLUSIONS

The proposed algorithm combines processing in the space, spectral and cepstral domains, taking advantage of the signal characteristics in each domain, and results in adequate identification of the blurring function parameter from noisy blurred images with SNR of down to 0dB, for motion blur, and 3dB for out-of-focus blur. In previous works this was achieved only with SNR of over 20dB.

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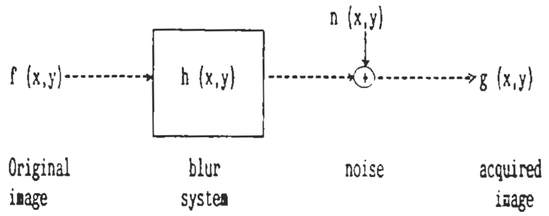

Fig. 1 - Image acquisition process

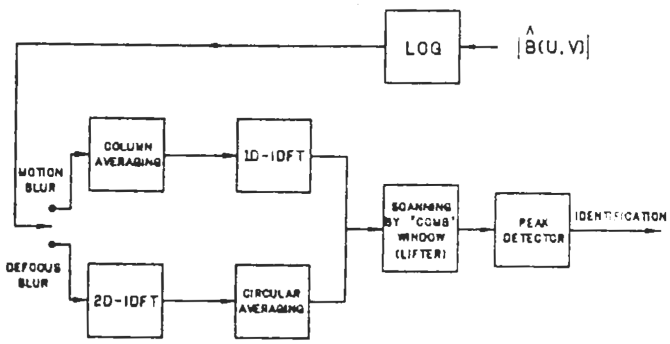

FIG 3b BLOCK DIAGRAM OF PROPOSED ALGORITHM - IDENTIFICATION IN THE CEPSTRAL DOMAIN STAGE

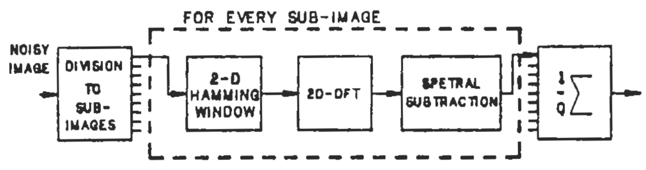

FIG.2 COMMONLY USED APPROACH OF SPECTRAL SUBTRACTION IMPLEMENTATION

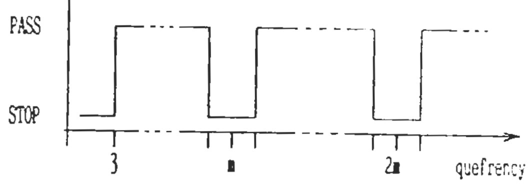

Fig. 4 - An adaptive "comb-window" (lifter) for identification in the cepstral domain

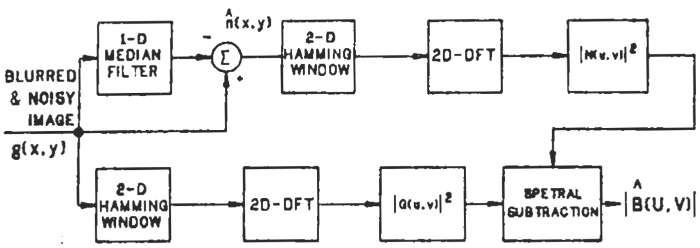

FIG.3a BLOCK DIAGRAM OF PROPOSED ALGORITHM - NOISE REDUCTION STAGE