

Signal Synthesis from Modified Discrete Short-Time Transform

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Abstract—The discrete short-time transform (DSTT) is a generalization of the discrete short-time Fourier transform (DSTFT). The necessary and sufficient conditions on the analysis filter, under which perfect reconstruction of the input signal is possible (when the DSTT is not modified), are presented. Furthermore, the class of linear modifications for which the original input can be reconstructed, when the modification is applied, is characterized. This characterization is useful, for example, in speech scrambling applications. The synthesis of an optimal (in the minimum mean-square-error sense) signal from a modified DSTT (MDSTT) of finite duration is presented. It is shown that for an analysis filter length which does not exceed a given value, the optimal synthesis scheme is independent of the duration of the given MDSTT, and is an extension of the weighted overlap add (WOLA) synthesis method. For longer analysis filters, the optimal synthesis scheme becomes quite cumbersome, and therefore, a steady-state solution (as the duration of the MDSTT approaches infinity) is presented for this case. It is further shown that this solution can be approximated with arbitrarily small reconstruction error, using a WOLA synthesis followed by an appropriate set of FIR correction filters.

I. INTRODUCTION

THE DSTFT is an important tool in analyzing and modifying speech signals [1]. It has been extensively used in analysis/synthesis (A/S) systems which contain a certain desired modification of the DSTFT. Typical applications are speech enhancement by spectral subtraction [2], time and frequency domain scaling [3], [5], speech scrambling by frequency permutations [4], and reconstruction of speech signals from the magnitude of their DSTFT [5].

Although, usually, the DFT is used in these systems, some contain other transforms (e.g., the DCT [9] and the GDFT [7]), and therefore, in this work we use the notion of DSTT in a unified framework for all those systems.

The most common synthesis scheme in these A/S systems is the WOLA method (cf. [6] and [7]). For this synthesis method, the necessary conditions on the *analysis and synthesis filters* under which an identity (or unity) system exists were presented in [1], where an identity system is an A/S system which reconstructs its original input signal without error when no modification is applied. However, since the WOLA synthesis method is not nec-

essarily the optimal approach, it seems worthwhile to obtain the necessary and sufficient conditions on the *analysis filter* under which an identity system exists, when an *arbitrary* synthesis scheme is allowed. Such conditions have not yet been presented, although some results in this direction appear in [10].

In the next section we present these conditions, and further discuss the conditions under which a *finite duration input signal* can be reconstructed without an error from its *finite duration* DSTT. These systems are denoted as finite-time identity systems (FTIS). We further present there the conditions for the existence of a transform-domain identity system (TDIS), which corresponds to a system for which its *synthesis stage* has the property that for every MDSTT it results in a time domain signal whose DSTT coincides with the given MDSTT.

It is also shown that when the decimation factor (R) is less than the transform size (M), there exists no TDIS. This implies that, in general, signal synthesis from MDSTT cannot be errorless. Therefore, in the transmitter-receiver pair based on two A/S systems shown in Fig. 1, the output signal from the receiver is not equal, in general, to the original input signal at the transmitter. This result was already noted in [10], when considering scrambling schemes based on A/S systems. However, when a *specific* modification is used, it might result in a *restricted* class of MDSTT for which an errorless synthesis scheme exists.

In Section III we consider a class of linear modifications of the DSTT, which is commonly used in scrambling applications [4], [10]. We characterize the subclass of these modifications for which there exists a system as in Fig. 2, that reconstructs the original input signal without error, and denote them as legal modifications (LM).

For *nonlinear* or *time-varying modifications*, and even for linear time-invariant modifications which are not LM, the issue of optimal synthesis arises. Some results in this direction were reported in [5], [8], and [10]. The optimization criterion in these works (which we also adopt here) is the minimization of the mean-square-error (MSE) between the DSTFT of the synthesized signal and the given MDSTFT. The main limitation in the result of [5] is in the nonrealistic assumption made that the *continuous* modified short-time Fourier transform is known instead of its discrete representation. As shown in [8], the synthesis scheme of [5] (which coincides with the WOLA

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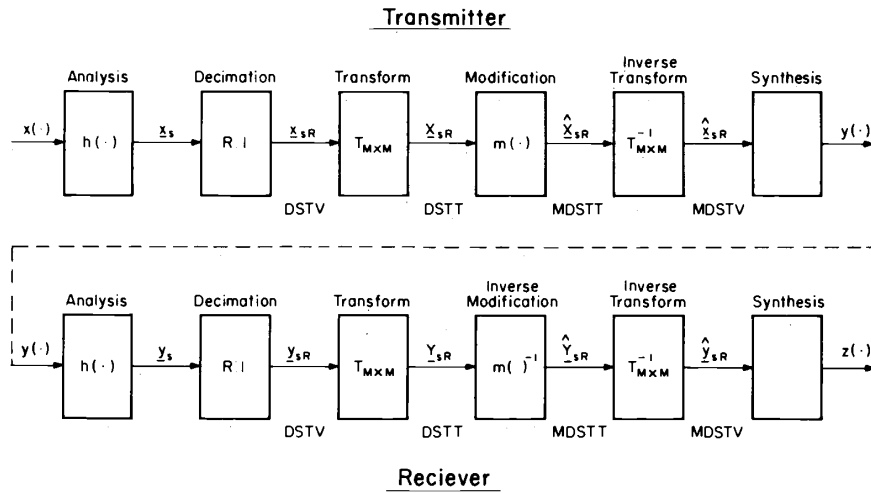


Fig. 1. A transmitter-receiver pair based on two A/S systems, the first contains a modification and the second contains its inverse.

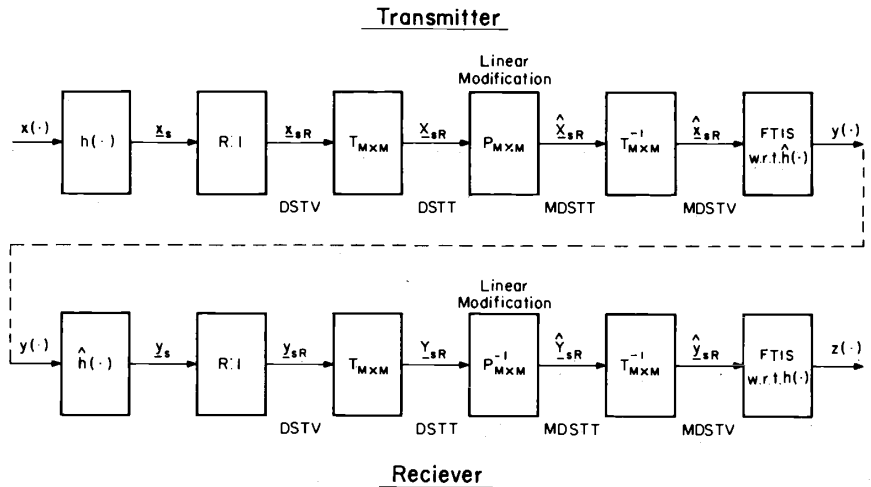


Fig. 2. A transmitter-receiver pair with linear modifications, FTIS synthesis systems, and possibly two different analysis systems.

method) is also optimal for the synthesis from MDSTFT provided that the transform-size M is greater or equal to the length of the analysis FIR filter. However, for an improved frequency resolution, one must use analysis filters which are longer than the transform size (cf. [7]). In this case, the optimal synthesis was regarded as an open problem, except for the special example of systems with no decimation at all (cf. [10]).

In Section IV we define the problems of optimal signal synthesis from a finite duration MDSTT, for analysis filters for which finite-time identity systems (FTIS) exist. It is shown there that the *unique* solution of this problem is obtained by a WOLA synthesis, followed by a "correction" system (CS). This system corresponds to solving M sets of linear equations, with overall dimension which equals the number of samples in the output signal. Al-

though the coefficients of these linear equations are independent of the given MDSTT, the size of these sets of equations can typically be in the thousands. Therefore, in general, the task of solving these equations is highly complex. As of now we have not restricted the analysis filter to be an FIR filter. However, when this assumption is made, the matrices that represent these systems of linear equations are banded matrices (i.e., have few nonzero diagonals), and an efficient implementation of the CS for this case is given. If L_h denotes the length of the analysis filter, then the complexity per output sample of this implementation is only $O((L_h/M))^2$ and is independent of the time duration of the given MDSTT. Similarly, the memory requirements per output sample are only $O(L_h/M)$ samples.

It is also shown, in Section IV, that for $L_h \leq RM/$

$\gcd(R, M)$ (where $\gcd(\cdot, \cdot)$ denotes the greatest common divisor, and R is the decimation factor), the correction system mentioned above is represented by block-diagonal matrices of linear equations, and therefore has a very simple recursive implementation. This implementation coincides with the result of [5] and [8] for $L_h \leq M$, and is its natural generalization.

For values of L_h larger than $RM/\gcd(R, M)$, a steady-state stable solution for an MDSTT of infinite duration is derived in Section V. Although this solution contains a noncausal IIR system, it is shown there that it can be approximated with arbitrarily uniformly small reconstruction error by a set of FIR correction filters. For the special example of systems without decimation ($R = 1$), this approximation coincides with the solution presented in [10].

Conclusions are drawn in the last section, and the details of the derivations are given in the Appendixes.

II. IDENTITY SYSTEMS

Before presenting the conditions for existence of the various types of identity systems, we shall define both the DSTT and the notion of a synthesis system.

The input to the analysis stage is a causal sequence denoted by $x(n)$ or $\{x(n)\}_{n=0}^{\infty}$. Its multiplication by the sliding window sequence $h(n)$ results in a causal sequence of M -dimensional vectors $\{x_{sR}\}_{s=1}^{\infty}$ which will be denoted in the sequel as discrete short-time vectors (DSTV), where the n th element of x_{sR} (the s th vector) is

$$x_{sR}(n) = \sum_{r=0}^{\infty} h(sR - (n + rM)) x(n + rM);$$

$$\infty > s \geq 1, \quad M - 1 \geq n \geq 0. \quad (1)$$

The analysis window $h(n)$ is a causal sequence assumed to have the property $h(0) = 0$ (which simplifies the presentation). The decimation factor (R) is assumed to be no greater than the transform size (M).

For the generality of the results, we assume throughout that all the signals are complex valued. Our results remain valid even when all the signals are real valued, as one should simply omit the complex conjugate sign * wherever it appears.

The DSTT is obtained from the DSTV by applying an arbitrary linear transform (represented by an $M \times M$ -dimensional regular matrix T) on each of the vectors in the DSTV sequence, i.e.,

$$X_{sR} = T x_{sR}; \quad \infty > s \geq 1. \quad (2)$$

In particular, for the matrix T whose (k, n) th element is $(1/\sqrt{M})e^{-j(2\pi/M)kn}$, the DSTT as defined in (1) and (2) is essentially the *discrete short-time Fourier transform (DSTFT)* (cf. [1] and [7]). The analysis window $h(n)$ can then be interpreted as the unit sample response of the *prototype filter* in the *analysis filter bank* (cf. [7]). For this reason we also denote $h(n)$ in the sequel as the analysis filter. Since $h(n)$ is typically a finite duration sequence, the summation in (1) is essentially finite, and the DSTT is well defined for any causal input sequence.

In the framework of DSTFT, the DSTV are the result of time-domain aliasing of the weighted [by $h(n)$] input sequence. This aliasing is due to the discretization (sampling) of the frequency variable in the short-time transform. When considering a general DSTT, the sampling in the transform domain should be represented by an appropriate transformation matrix, instead of the restricted form used in (1) and (2). All the results in this section, as well as some of the results derived in the sequel, can be restated under this more general framework, at the expense of more complicated expressions and lengthier derivations. Bearing in mind that most of the useful DSTT's are still based on the Fourier transform (e.g., the DCT and GDFT mentioned earlier), we prefer to limit the scope of this paper to DSTT's for which (1) and (2) make sense. The interested reader can use this work as a guideline for performing generalizations.

The synthesis system reconstructs a causal time-domain sequence from a given MDSTT, and without loss of generality we can make the following assumptions.

a) The output of the system [which is denoted by $y(n)$] is well defined for the given MDSTT (denoted by the sequence of vectors $\{X_{sR}\}_{s=1}^{\infty}$).

b) Each sample of $y(n)$ depends only on a finite (arbitrarily long) prefix¹ of the MDSTT, and thus can be obtained with *finite delay*.

Now, when we consider the existence of identity systems, we assume that all the parameters regarding the DSTT (i.e., the values of M and R , the filter $h(n)$, and the transform T) *have already been chosen*. Therefore, it is convenient (although not common) to define an identity system with respect to these parameters as the synthesis system that has the desired property when it is combined with the *given* analysis system. We distinguish between three types of identity systems, as follows.

Definition 1: An identity system (IS) is a *synthesis* system which reconstructs the original input sequence when no modification is applied, i.e.,

$$(\forall s \geq 1)(X_{sR} = \hat{X}_{sR}) \Rightarrow (\forall n \geq 0)(y(n) = x(n)). \quad (3)$$

Definition 2: A finite-time identity system (FTIS) is an IS having the additional property that for every $L \geq 1$, the first ILR samples of $y(n)$ depend only on the first IL vectors of the MDSTT.²

Note: Throughout we denote $\gcd(R, M)$ by g and define $I \triangleq M/g$, $J \triangleq R/g$.

Definition 3: A transform-domain identity system (TDIS) is a *synthesis* system which generates for every MDSTT a causal sequence $y(n)$ whose DSTT coincides with the given MDSTT.

The following propositions summarize the results regarding existence of the various types of identity systems.

¹A finite prefix of the MDSTT denotes a finite sequence of vectors $\{X_{sR}\}_{s=1}^{s_0}$ with $s_0 < \infty$.

²Note that there is *no* correspondence between the concept of FTIS and the use of FIR analysis (synthesis) filters.

The derivations (which are constructive, i.e., contain closed-form examples of these systems) are presented in Appendix A.

Proposition 1: The existence of the various identity systems is independent of the specific transform used in the analysis.

Proposition 2: There exists a TDIS iff $R = M$ and $h(n) \neq 0$ at least for $M \geq n \geq 1$.

Proposition 3: There exists an FTIS iff $h(n) \neq 0$ at least for $R \geq n \geq 1$.

Proposition 4: Every IS is a linear system.

Proposition 5:

a) There exists an IS only if *none* of the R polyphases (cf. [7]) of the analysis filter $\{h(p + Rk)\}_{k=0}^{R-1}$, $R \geq p \geq 1$ is identically zero.

b) When $J = 1$ (i.e., R divides M), this condition is also a sufficient condition for the existence of an IS.

c) There exists an IS iff for every $M \geq p \geq 1$, there exists a finite prefix of the DSTV from which the J samples $\{x(Mr - p)\}_{r=1}^J$ of the input signal can be uniquely determined.

The conditions of propositions 2, 3, and 5 are valuable in the design of analysis systems and some of them are also used in the next sections.

III. LEGAL LINEAR MODIFICATIONS

We consider now a transmitter-receiver pair as in Fig. 1 but with linear modification of the DSTT vectors, represented by an $M \times M$ nonsingular matrix P , i.e.,

$$\hat{X}_{sR} = P X_{sR}; \quad \infty > s \geq 1 \quad (4a)$$

$$\hat{Y}_{sR} = P^{-1} Y_{sR}; \quad \infty > s \geq 1. \quad (4b)$$

We introduce another degree of freedom by allowing the analysis filter of the receiver $\hat{h}(n)$ to be different from the analysis filter $h(n)$ of the transmitter.

Definition 4: A legal modification (LM), with respect to the analysis system of the transmitter, is a matrix P for which there exists a synthesis system for the transmitter, and an analysis filter $\hat{h}(n)$ for the receiver, such that

$$(\forall s \geq 1)(\hat{X}_{sR} = P X_{sR}) \Rightarrow (\forall s \geq 1)(Y_{sR} = \hat{X}_{sR}). \quad (5)$$

For simplicity we shall further assume throughout this section that there exists an FTIS with respect to the analysis filter of the transmitter.

The following proposition is derived in Appendix B.

Proposition 6: If P is an LM with respect to the analysis system of the transmitter, using $\hat{h}(n)$ as the analysis filter of the receiver, then

a) there exists an FTIS with respect to this analysis filter, and

b) the transmitter-receiver pair depicted in Fig. 2 guarantees errorless reconstruction of the original input of the transmitter, i.e., $z(n) = x(n)$.

This proposition explains why the use of LM is recommended, and why the characterization of the subclass

of LM's is an important issue. For example, the problems of low quality reconstruction in a scrambling system (cf. [4]) of the type presented in Fig. 2 (that were reported in [10]) disappear once one restricts oneself to using the subclass of LM's.

As we show below, a modification might be an LM with respect to one analysis filter, but not so with respect to another analysis filter. It is therefore more convenient in the design of the system to use only matrices which are LM with respect to any analysis filter to which there exists an FTIS. Therefore, we have Definition 5 as follows.

Definition 5: A universal legal modification (ULM) is a matrix P which is an LM with respect to any analysis filter for which there exists an FTIS.

The following five propositions, which are derived in Appendix B, summarize the characterization of LM's and ULM's.

Proposition 7: The parameters M, R , the analysis filter $h(n)$, and the locations of nonzero elements of the matrix, $\hat{P} \triangleq (T^{-1}PT)$, are sufficient in order to determine whether or not P is an LM.

Proposition 8: A matrix \hat{P} represents an LM *only if* \hat{P} is a block-diagonal matrix with blocks of dimensions $g \times g$ each [where $g \triangleq \gcd(R, M)$].

Proposition 9: For $R = M$, and $\hat{h}(n) = h(n)$, every regular matrix P is a ULM.

Proposition 10: For $R < M$, and $\hat{h}(n) = h(n)$, there exists an FIR analysis filter, for which every block-diagonal matrix \hat{P} with blocks of dimensions $g \times g$ (each) represents an LM.

Proposition 11:

a) For $R < M$, a ULM is characterized by

$$\hat{P} = \Lambda \begin{bmatrix} \pi & 0 & \cdots & 0 \\ 0 & \pi & & \vdots \\ \vdots & \vdots & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & \pi \end{bmatrix} \quad (6)$$

where Λ is any nonsingular diagonal matrix, and π is any $g \times g$ -dimensional permutation matrix.

b) There exists an FIR analysis filter $h(n)$ of length $2R$ samples, such that every LM with respect to $h(n)$ is a ULM.

Note: For scrambling, one usually uses the DFT with a permutation matrix P (cf. [4] and [10]). In general, the resulting \hat{P} does not represent an LM (it usually contradicts the conditions stated in proposition 8 above). However, it is easily verified that any circulant permutation (i.e., of the form $P_{ij} = \delta(j \equiv (i + a) \pmod{M})$, with $M - 1 \geq a \geq 0$) is a ULM for the DFT transform. Thus, if one restricts oneself to using only these M permutations, then reconstruction of the original input without error is guaranteed.

Some of the ideas used in this section for linear modifications can be extended to invertible nonlinear modifications. In particular, the definitions of LM and ULM are easily extended as well as part b) of proposition 6 (the

errorless reconstruction property) and proposition 9. However, unlike the linear case, it is hard, in general, to obtain explicit characterization of the LM's and ULM's (the extensions of propositions 8, 10, and 11).

IV. OPTIMAL SIGNAL SYNTHESIS FROM FINITE DURATION MDSTT

Assume that an arbitrary MDSTT of finite duration $\{\hat{X}_{sR}\}_{s=1}^{IL}$ is given. In general, there might be no causal sequence $y(n)$ whose DSTT coincides with this MDSTT. We restrict the discussion to MDSTT durations which are integer multiples of $I = M/\text{gcd}(R, M) = M/g$, merely for simplicity of the presentation.

We note that the first IL vectors of the DSTT depend only on $\{y(n)\}_{n=0}^{ILR-1}$ and, therefore, define the *optimal* (in the MSE sense) *signal* as the finite sequence $y(n)$ ($\{y(n)\}_{n=0}^{ILR-1}$) that minimizes the following error measure:

$$D_L(y(n)) \triangleq \frac{1}{IL} \sum_{s=1}^{IL} (\hat{X}_{sR} - Y_{sR})^* (\hat{X}_{sR} - Y_{sR}) \quad (7)$$

where the asterisk denotes the complex conjugate-transpose operation.

Before proceeding with the solution of this problem, we make two reasonable assumptions regarding the analysis system, as follows.

Assumption 1: There exists an FTIS with respect to the analysis filter $h(n)$ and the parameters M, R (i.e., $h(n) \neq 0$ at least for $M \geq R \geq n \geq 1$, as required in proposition 3).

Assumption 2: The transform T is a unitary transform (i.e., T^*T is the identity matrix).

Due to assumption 1 it is guaranteed that for $\hat{X}_{sR} = X_{sR}$ (i.e., the MDSTT is actually a DSTT), the optimal signal $y(n)$ will coincide with the prefix of the original input signal $x(n)$, and therefore, the *optimal synthesis scheme is an FTIS with respect to the analysis system*.

Due to assumption 2, the error measure $D_L(y(n))$ can be restated in terms of the MDSTV generated from the given MDSTT and the DSTV of the sequence $y(n)$ as follows:

$$D_L(y(n)) = \frac{1}{IL} \sum_{s=1}^{IL} (\hat{x}_{sR} - y_{sR})^* (\hat{x}_{sR} - y_{sR}) \quad (8)$$

and therefore, except for the generation of the MDSTV from the given MDSTT using the matrix T^{-1} , the optimal synthesis scheme is independent of the specific transform used in the analysis.

When the modification is a known linear modification represented by the matrix P (which is in general not an LM), one can define a slightly different error measure as

$$\hat{D}_L(y(n)) \triangleq \frac{1}{IL} \sum_{s=1}^{IL} (X_{sR} - P^{-1}Y_{sR})^* (X_{sR} - P^{-1}Y_{sR}). \quad (9)$$

However, for *unitary* linear modifications (which are the

typical ones), this error measure coincides with $D_L(y(n))$.

The results presented below can be extended to the case of nonunitary transform and arbitrary positive definite (P.D.) error weighting matrix, but the derivations are much more tedious (cf. [13]).

Since the DSTV is a linear function of the time signal, it follows from (8) that $D_L(y(n))$ is a quadratic form in terms of the unknown samples of $y(n)$, and since $D_L(y(n)) \geq 0$ for every sequence $y(n)$, this is a non-negative definite quadratic form. Moreover, for the specific case of $\hat{X}_{sR} = X_{sR}$, we have already noticed that due to assumption 1, there exists a unique optimal signal $y(n)$ [which is the prefix of $x(n)$]. This directly implies that $D_L(y(n))$ is a P.D. quadratic form, and therefore, *there always exists a unique optimal signal*, which is the solution of the linear set of equations obtained from $\nabla D_L(y(n)) = \mathbf{0}$.

Substituting (1) in (8) and calculating the gradient equations, we obtain

$$\begin{aligned} & \sum_{s=1}^{IL} y_{sR}((k)_M) h^*(sR - k) \\ &= \sum_{s=1}^{IL} \hat{x}_{sR}((k)_M) h^*(sR - k); \\ & ILR - 1 \geq k \geq 0. \end{aligned} \quad (10)$$

The right-hand side of (10) corresponds to WOLA synthesis of the given MDSTT with the noncausal synthesis filter: $f(n) = h^*(-n)$, $\infty > n > -\infty$.

The left-hand side of (10) corresponds to what we denoted earlier as the correction system (CS). Substituting (1) in (10) and rearranging the resulting expressions, we obtain

$$\begin{aligned} & \sum_{n=1}^{LJ} W_L^{(p)}(k, n) y^{(p)}(n) = u^{(p)}(k); \\ & LJ \geq k \geq 1, M \geq p \geq 1 \end{aligned} \quad (11a)$$

where

$$u^{(p)}(k) \triangleq \sum_{s=1}^{IL} \hat{x}_{sR}(M - p) h^*(sR + p - kM) \quad (11b)$$

$$y^{(p)}(k) \triangleq y(KM - p) \quad (11c)$$

and

$$\begin{aligned} & W_L^{(p)}(k, n) \\ & \triangleq \sum_{s=1}^{IL} h^*(sR + p - kM) h(sR + p - nM). \end{aligned} \quad (11d)$$

Since the M subsequences $u^{(p)}(\Delta)$ are the output of the WOLA synthesis method, the CS involves solving M sets of LJ linear equations each (recall, $J = R/g$). The complexity of a brute-force solution is $\mathcal{O}(M(LJ)^3)$, and this is an enormous amount of computations for typical values of $L \geq 1000$.

The discussion so far was for any analysis filter. However, for *FIR analysis filter*, reasonable implementation of the CS is possible. Therefore, we introduce now the third assumption on the analysis system.

Assumption 3: The analysis filter $h(n)$ is an FIR of length $L_h \triangleq MJK_h$. There is essentially no restriction in choosing L_h as an integer multiple of MJ , since zero padding can always be used, it only simplifies the derivations below.

Under assumption 3 we make the following two observations.

a) Causality of the WOLA synthesis that implements (11b) can be guaranteed by incorporating a finite delay of L_h samples in the synthesis scheme.

b) When the matrix $W_L^{(p)}$ is arranged as an $L \times L$ matrix with entries which are $J \times J$ matrices, then

1) it is a banded matrix, where the $(2K_h - 1)$ main diagonals are the only nonzero diagonals; and

2) except for the last K_h rows and columns, it is a Toeplitz-Hermitian matrix, with its entries being independent of the value of L .

This observation is based on the analysis of (11d) under assumption 3, and can be summarized formally as

$$W_L^{(p)} = \begin{bmatrix} B_o^{(p)} & \cdots & B_{(K_h-1)}^{(p)} & 0 & \cdots & 0 \\ \vdots & & & & & \\ B_{-(K_h-1)}^{(p)} & & & & & \\ 0 & & B_o^{(p)} & & B_{(K_h-1)}^{(p)} & 0 \\ \vdots & & & & & \\ \vdots & & & & B_{-(K_h-1)}^{(p)} & C^{(p)} \\ 0 & \cdots & 0 & & & \end{bmatrix} \quad (12a)$$

where, for $|\Delta| \leq (K_h - 1)$,

$$\begin{aligned} B_{\Delta}^{(p)}(i, j) &= \sum_{s=1}^{IK_h + (I-1)} h(sR + p - jM) h^*(sR + p - iM + \Delta MJ); \\ J &\geq i, j \geq 1 \end{aligned} \quad (12b)$$

and

$$\begin{aligned} C^{(p)}(k, n) &= \sum_{s=1}^{IK_h} h(sR + p - nM) h^*(sR + p - kM); \\ K_h J &\geq k, n \geq 1. \end{aligned} \quad (12c)$$

Efficient On-Line Implementation of the CS for $L_h \leq MJ$: For $L_h \leq MJ$, then $K_h = 1$, and therefore, $W_L^{(p)}$ is a block diagonal matrix (with blocks of size $J \times J$) in which all the blocks equal to $B_o^{(p)}$, except for the last block

which equals $C^{(p)}$. Therefore, $[W_L^{(p)}]^{-1}$ is obviously given by

$$[W_L^{(p)}]^{-1} = \text{diag} [B_o^{(p)-1}, \dots, B_o^{(p)-1}, C^{(p)-1}]. \quad (13)$$

Thus, the CS design involves inversion of $2M$, nonsingular $J \times J$ matrices, and the optimal synthesis is implemented as depicted in Fig. 3. In the special case of either $J = 1$ or $L_h \leq M$, the matrices $B_o^{(p)}$ become diagonal matrices and the optimal synthesis coincides (except for boundary conditions) with the WOLA synthesis obtained in [5] and [8].

Efficient Batch Implementation of the CS for $L_h > MJ$: For $L_h > MJ$, then $K_h > 1$ and $W_L^{(p)}$ is not a block diagonal matrix. However, it is still a banded matrix, and this property is utilized to derive an efficient batch implementation of the CS. This implementation holds only under the following additional assumption on the analysis system.

Assumption 4: The $J \times J$ matrix $B_{(K_h-1)}^{(p)}$ is a nonsingular matrix, for any $1 \leq p \leq M$. However, for the important case of $J = 1$ (i.e., R divides M), this assumption

is essentially unneeded, since $B_{(K_h-1)}^{(p)}$ is a scalar, and if $B_{(K_h-1)}^{(p)} = 0$ for some value of p , we can easily adapt the implementation presented below to this case by using $B_{(K_h-2)}^{(p)}$ instead of $B_{(K_h-1)}^{(p)}$, and so on.

We now regard the sequences $y^{(p)}(n)$ and $u^{(p)}(n)$ as sequences of L vectors in \mathbb{C}^J (denoted as $y_n^{(p)}$ and $u_n^{(p)}$, respectively) by combining every J consecutive samples into one vector. Thus, the $(L - K_h)J$ first equations of (11a) are [due to (12a)]

$$\sum_{k=-(K_h-1)}^{(K_h-1)} B_k^{(p)} y_{(n+k)}^{(p)} = u_n^{(p)}; \quad (L - K_h) \geq n \geq 1 \quad (14)$$

where $y_n^{(p)} \triangleq \mathbf{0}$ for $n \leq 0$.

Under assumption 4 we rewrite (14) as an (matrix) IIR

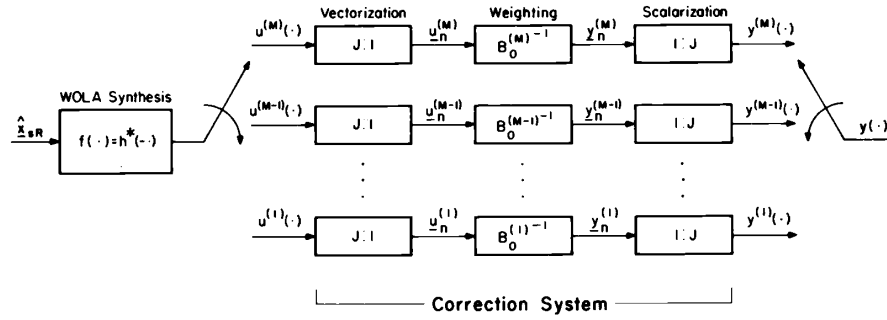


Fig. 3. An on-line implementation of the optimal synthesis scheme for analysis filter length $L_h \leq MJ$.

filter

$$y_n^{(p)} = [B_{(K_h-1)}^{(p)}]^{-1} u_{(n-(K_h-1))}^{(p)} - \sum_{k=1}^{2(K_h-1)} [B_{(K_h-1)}^{(p)}]^{-1} \cdot B_{(K_h-1-k)}^{(p)} y_{(n-k)}^{(p)}; \quad (L-1) \geq n \geq K_h. \quad (15)$$

Therefore, the original solution of (11a) (except for the last J samples) can be described, according to (15), as the sum of the particular solution and a linear combination of $(K_h - 1)J$ homogeneous solutions.

The particular solution is obtained by using (15) with zero initial conditions ($y_n^{(p)} = \mathbf{0}$ for $(K_h - 1) \geq n \geq 1$), and the (m, i) th element of the basis of the homogeneous solution space (where $(K_h - 1) \geq m \geq 1, J \geq i \geq 1$) is obtained by using (15) with zero input ($u_n^{(p)} = \mathbf{0}$), and initial conditions $y_n^{(p)} = \mathbf{0}$ for $n \neq m$, and $y_m^{(p)}$ is the i th unit vector in \mathbb{C}^J .

The calculation of the particular solution and the basis of the homogeneous solution requires $\mathcal{O}(K_h^2 J^2)$ operations per output sample of the CS. Given these $(K_h - 1)J + 1$ solutions, their appropriate linear combination is determined by solving numerically the last JK_h equations in (11a) which involve the matrices $C^{(p)}$. This is a *batch implementation* since before we obtain all the samples of $u^{(p)}(n)$, we cannot determine any of the samples of $y^{(p)}(n)$. However, it requires a small amount of operations per output sample which is independent of the duration of the given MDSTT, and a small (fixed) amount of temporary memory per output sample (i.e., $\mathcal{O}(1 + (K_h J/M))$ samples).

However, for large values of L , stability problems may arise, and as we shall show in the next section—it is preferred to use a steady-state on-line, nonrecursive, and stable implementation of the CS presented there.

V. THE STEADY-STATE STABLE OPTIMAL SYNTHESIS

There are two different approaches in which an on-line implementation of the CS can be derived. Although the justifications of these derivations differ, their final results coincide.

The first approach, which we only briefly review here

without supplying the proofs, is an extension of the algebraic approach of [10], presented there for $R = J = 1$. It is based on finding a sequence of matrices $\{\tilde{W}_L^{(p)}\}_{L=K_h}^{\infty}$ which are asymptotically equivalent (cf. [12]) to $\{W_L^{(p)}\}_{L=K_h}^{\infty}$, and provides an analytical solution for $[\tilde{W}_L^{(p)}]^{-1}$.

For $R = 1$ it is claimed in [10], following the results of [12], that the sequence $\tilde{W}_L^{(p)}$ of circulant matrices with first row $(B_o^{(p)}, \dots, B_{(K_h-1)}^{(p)}, 0, \dots, 0, B_{-(K_h-1)}^{(p)}, \dots, B_{-1}^{(p)})$ is asymptotically equivalent to $W_L^{(p)}$. Therefore, $[\tilde{W}_L^{(p)}]^{-1}$ is also a circulant matrix, and its first row is obtained from $\{B_k^{(p)}\}_{k=-(K_h-1)}^{(K_h-1)}$ by a DFT of size L followed by replacement of each transform element value by its reciprocal, and an IDFT (cf. [11]). Since $\{\tilde{W}_L^{(p)}\}_{L=K_h}^{\infty}$ is asymptotically equivalent to $\{W_L^{(p)}\}_{L=K_h}^{\infty}$, it is suggested in [10] to implement an approximate CS by replacing $W_L^{(p)}$ with $\tilde{W}_L^{(p)}$ in (11a). Due to finite energy considerations, it is shown there that the first row of $[\tilde{W}_L^{(p)}]^{-1}$ can be "clipped" so that this matrix essentially represents FIR filtering of the sequence $u^{(p)}(n)$, and therefore the structure of the proposed CS is as depicted in Fig. 4. It can be easily verified that this CS can be embedded inside the WOLA synthesis.

From the discussion in the previous section it is clear that this solution holds whenever R divides M (i.e., $J = 1$), and not only for $R = 1$.

The generalization of the algebraic approach to the case of $J > 1$ is as follows.

a) Following the same line of proof as in [12] it can be shown that $\{W_L^{(p)}\}_{L=K_h}^{\infty}$ is asymptotically equivalent to $\{\tilde{W}_L^{(p)}\}_{L=K_h}^{\infty}$ which is now a block-circulant matrix with $J \times J$ blocks and first "row" $(B_o^{(p)}, \dots, B_{(K_h-1)}^{(p)}, 0, \dots, 0, B_{-(K_h-1)}^{(p)}, \dots, B_{-1}^{(p)})$.

b) If

$$\det \left\{ \sum_{k=-(K_h-1)}^{(K_h-1)} B_k^{(p)} z^{-k} \right\} \neq 0 \quad \text{for } |z| = 1, \quad (16)$$

then $[\tilde{W}_L^{(p)}]^{-1}$ exists, for every value of L , and is a block-circulant matrix with $J \times J$ blocks. The entries of the first "row" of $[\tilde{W}_L^{(p)}]^{-1}$ are obtained by J^2 DFT's of size L , which are done independently on the J^2 entries of the sequence of matrices $\{B_k^{(p)}\}_{k=-(K_h-1)}^{(K_h-1)}$, then a pointwise

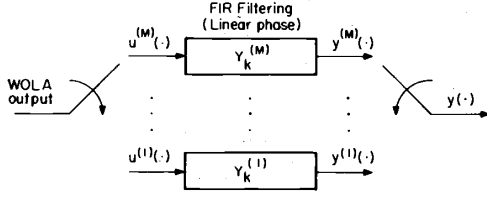


Fig. 4. An on-line implementation of the correction system for $J = 1$, which is the steady-state, stable solution.

matrix inversion in the transform domain is performed, followed by J^2 IDFT's of size L , done independently on each entry. This derivation is proved following the same lines as in [11].

c) The considerations presented in [10] can also be applied here [due to (16)], to justify the use of only a finite ("clipped") sequence of $J \times J$ matrices $\{Y_k^{(p)}\}_{k=-(T_\epsilon-1)}^{(T_\epsilon-1)}$ to approximate the operation of $[\tilde{W}_L^{(p)}]^{-1}$. For every small accuracy level required $\epsilon > 0$, there exists an integer $T_\epsilon \geq 1$ such that the approximation error is bounded below ϵ .

The suggested implementation (depicted also in Fig. 5) is according to

$$y_n^{(p)} = \sum_{k=-(T_\epsilon-1)}^{(T_\epsilon-1)} Y_k^{(p)} u_{n-k}^{(p)}; \quad \infty > n \geq 1 \quad (17)$$

where $u_n^{(p)} \triangleq \mathbf{0}$ for $n < 1$. This is an *on-line* implementation of the CS, with a reconstruction delay of $(T_\epsilon - 1)$ MJ samples. However, it is only an approximation of the solution of (11a). The second approach, with which we shall prove this result, also indicates the type of approximation done in (17).

As L approaches infinity, the submatrix $C^{(p)}$ is eliminated from $W_L^{(p)}$, and (11a) can be regarded as the following deconvolution problem.

Given the sequence $\{u_n^{(p)}\}_{n=1}^\infty$, find a sequence $\{y_n^{(p)}\}_{n=1}^\infty$ such that

$$\sum_{k=-(K_h-1)}^{(K_h-1)} B_{-k}^{(p)} y_{n-k}^{(p)} = u_n^{(p)}; \quad \infty > n \geq 1 \quad (18)$$

where $y_n^{(p)} \triangleq \mathbf{0}$ for $n < 1$.

It is easily verified that in general (18) has an infinite number of solutions [one for every choice of $\{y_n^{(p)}\}_{n=1}^{(K_h-1)}$ according to (15)]. However, since $B_k^{(p)*} = B_{-k}^{(p)}$ as is easily verified in (12b), the zeros of the determinant in the left-hand side of (16) are arranged in pairs, where for every zero inside the unit circle there exists a corresponding zero outside the unit circle. Because of this property most of the solutions of (18) are *unstable*. The following proposition is proved in Appendix C regarding the stable solution.

Proposition 12: The only stable solution of (18) [under the condition of (16)], except for a possible mismatch with

$u_n^{(p)}$ for $(K_h - 1) \geq n \geq 1$, is given by

$$y_n^{(p)} = \sum_{k=-\infty}^{\infty} Y_k^{(p)} u_{n-k}^{(p)}; \quad \infty > n \geq 1 \quad (19)$$

where $u_n^{(p)} \triangleq \mathbf{0}$ for $n < 1$, and the sequence of matrices $\{Y_k^{(p)}\}_{k=-\infty}^\infty$ is obtained from $\{B_{-k}^{(p)}\}_{k=-(K_h-1)}^{(K_h-1)}$ by a (continuous frequency) Fourier transform, followed by matrix inversion and inverse transform.

Stability is defined in the following sense:

$$u_n^{(p)} = \mathbf{0} \quad \text{for } n \geq n_o = > \lim_{n \rightarrow \infty} \{y_n^{(p)*} y_n^{(p)}\} = 0. \quad (20)$$

The solution given by (19) is denoted as the *stable steady-state solution*, since we ignore a mismatch in the first $(K_h - 1)$ equations.

It is also shown in Appendix C that the sequence $Y_k^{(p)}$ can be "clipped" as follows.

Proposition 13:

a) For every value of $\epsilon > 0$, there exists an integer $T_\epsilon \geq 1$ such that the *error* in each output sample between the solutions using (17) and (19) is below ϵ .

b) For every value of $\epsilon > 0$, there exists an integer Q_ϵ [with $Q_\epsilon \geq \max(2T_\epsilon - 1, 2K_h - 1)$], such that the evaluation of $\{Y_k^{(p)}\}_{k=-(T_\epsilon-1)}^{(T_\epsilon-1)}$ using Q_ϵ -dimensional DFT's, instead of the continuous frequency Fourier transform, introduces a reconstruction error which is *uniformly bounded below* ϵ .

Under mild conditions, the values of T_ϵ and Q_ϵ are independent of the sequence $\{u_n^{(p)}\}_{n=1}^\infty$, and therefore can be determined *a priori* in the design of the CS.

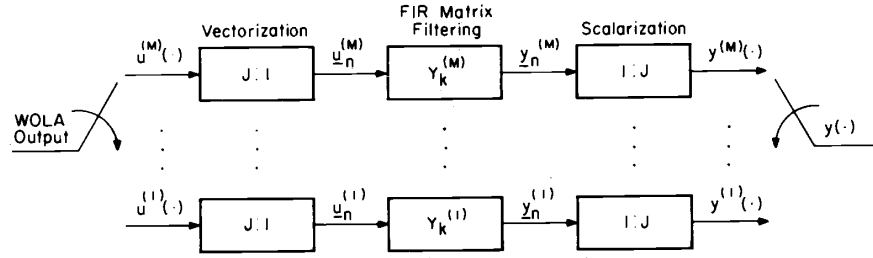
This proposition links the *on-line* solution (of Fig. 5) which results by using the extended algebraic approach, with the only *steady-state stable* solution of the deconvolution problem in (18). It also gives certain bounds on T_ϵ and Q_ϵ , but they are only of theoretical value.

When using this implementation it is suggested to choose T_ϵ and Q_ϵ according to the allowed complexity of the synthesis system, and to test empirically the accuracy of the synthesis by comparing two or three different choices of (T_ϵ, Q_ϵ) .

VI. CONCLUSIONS

In this paper we studied three subjects related to the synthesis of a signal from a modified discrete short-time transform (MDSTT). These are identity systems, legal linear modifications, and optimal synthesis. The main contributions are as follows.

a) New necessary and sufficient conditions on the analysis filter which guarantee the existence of identity systems are presented. The consequences of these conditions are that every identity system has to be linear, and that there exists a transform domain identity system only when the decimation rate (R) equals the transform size (M). The most useful result from the practical point of

Fig. 5. An on-line implementation of the correction system for $J > 1$.

view is that a finite-time identity system exists iff none of the first R samples of the analysis filter is zero.

b) For systems of the type described in [4] and [10], legal modifications (LM) are defined, and it is shown that for these modifications the scheme depicted in Fig. 2 guarantees errorless reconstruction of the original input. Furthermore, the class of LM is characterized, as well as the subclass of ULM (i.e., those modifications which are LM with respect to any analysis filter with nonzero first R samples).

c) Given an arbitrary MDSTT sequence of finite duration, the optimal (in the mean-square-error sense) synthesis is defined. It is shown that the optimal signal synthesis is done by a WOLA system followed by a linear correction system (CS).

Efficient on-line implementation of the CS for FIR analysis filters with length which is not greater than MJ [with $J = R/\text{gcd}(R, M)$], is presented (e.g., in Fig. 3), and coincides, for some particular cases, with the results of [5] and [8].

For FIR analysis filters whose length is greater than MJ , an efficient *batch* implementation of the CS is presented. It involves a relatively small computational complexity and memory requirements per output sample, which are independent of the given MDSTT duration.

Moreover, for infinite duration MDSTT the *stable, steady-state on-line* implementation of the CS is presented (e.g., Figs. 4 and 5). This implementation coincides with the results of [10] for the particular case of $R = 1$ (no decimation in the DSTT), and leads to a relatively simple synthesis scheme.

We note here that whereas for analysis filters whose length is not greater than the transform size the optimal synthesis scheme is essentially a WOLA system (as mentioned also in [8]), *it is not so for longer analysis filters.*

As the optimal signal synthesis structure does not depend explicitly on the given MDSTT, all these schemes tolerate synchronization and channel delays. They will simply cause the same delay in the reconstructed signal.

APPENDIX A

Proof of Proposition 1: Since the transform T is a linear one-to-one and onto operator that transfers a finite prefix of the DSTV into a prefix of the same length of the DSTT, the proposition follows from definitions 1-3.

Remark: Due to Proposition 1, we prove propositions 2-5 for $T = I$ without loss of generality.

Proof of Proposition 2:

a) *Necessity of the TDIS Conditions:* Consider $x_{sR}(n)$ for $s = 1$. From (1), the causality of both $x(n)$ and $h(n)$, and the assumption that $R \leq M$, it follows that $x_R(n) = h(R - n)x(n)$ for $R - 1 \geq n \geq 0$ and zero for $M - 1 \geq n \geq R$. Therefore, if either $R < M$ or $h(k) = 0$ for some value of $R \geq k \geq 1$, then the vector x_R must have at least *one zero element* in it. Thus, in this case, for any given MDSTT in which all the elements of the first vector are nonzero, there is no causal sequence $y(n)$ whose DSTT coincides with the given MDSTT. This implies that no TDIS exists.

b) *Sufficiency of the TDIS Conditions:* Assume that $R = M$ and $h(n) \neq 0$ for $M \geq n \geq 1$. In this case (1) can be rewritten as

$$x_{sM}(n) = \sum_{t=0}^{s-1} h(M - n + Mt) x(n + (s - t - 1)M);$$

$$\infty > s \geq 1, M - 1 \geq n \geq 0. \quad (\text{A1})$$

Given an arbitrary MDSTT $\{\hat{X}_{sM}\}_{s=1}^{\infty}$, the following system is a TDIS (for $T = I$), as can be easily verified:

$$y(n + (s - 1)M)$$

$$\triangleq \frac{1}{h(M - n)} \left\{ \hat{X}_{sM}(n) - \sum_{t=1}^{s-1} h(M - n + Mt) \cdot y(n + (s - 1)M - tM) \right\};$$

$$\infty > s \geq 1, M - 1 \geq n \geq 0. \quad (\text{A2})$$

Proof of Proposition 3:

a) *Necessity of the FTIS Conditions:* Consider the reconstruction of the first IR samples of $x(n)$ from the vectors $\{x_{sR}\}_{s=1}^I$. The samples $\{x(n)\}_{n=(I-1)R}^{IR-1}$ affect only the vector x_{IR} as easily verified in (1). Moreover, each of these R samples affects a different (and only one) element of this vector. Since the n th sample is multiplied by $h(IR - n)$, it follows that if $h(k) = 0$ for $R \geq k \geq 1$, then $x(IR - k)$ does not affect x_{IR} at all, and therefore cannot be reconstructed from $\{x_{sR}\}_{s=1}^I$, i.e., there exists no FTIS.

b) *Sufficiency of the FTIS Conditions:* Assume that $h(n) \neq 0$ for $R \geq n \geq 1$. We shall inductively build an

FTIS. From x_R we reconstruct $\{x(n)\}_{n=0}^{R-1}$, by $x(n) = x_R(n)/h(R-n)$ $R-1 \geq n \geq 0$. Now, assume that $\{x(n)\}_{n=0}^{kR-1}$ was already reconstructed from $\{x_{sR}\}_{s=1}^k$, and then the reconstruction of $\{x(n)\}_{n=kR}^{(k+1)R-1}$ from $x_{(k+1)R}$ is by

$$x(n) = \frac{1}{h((k+1)R-n)} \left\{ x_{(k+1)R}((n)_M) - \sum_{r=1}^{\lfloor n/M \rfloor} h((k+1)R-n+Mr) x(n-Mr) \right\}. \quad (\text{A3})$$

This is obviously a well-defined FTIS.

Proof of Proposition 4: Assume an IS exists. We use it to reconstruct $x(n)$ from its DSTT X_{sR} , and to reconstruct $y(n)$ from its DSTT Y_{sR} . Consider the sequence $\{\alpha X_{sR} + \beta Y_{sR}\}_{s=1}^\infty$ with $\alpha, \beta \in \mathbb{C}$. Since the mapping from time domain sequences to DSTT is a linear mapping, it follows that this later sequence is the DSTT of $\alpha x(n) + \beta y(n)$. From definition 1, the use of the IS must lead to the output sequence $\alpha x(n) + \beta y(n)$, and thus we have showed that any IS is linear.

Proof of Proposition 5:

Part a): If $h(p_o + Rk) = 0$ for every $k \geq 0$ and some $R \geq p_o \geq 1$, then, for example, the sample $x(R - p_o)$ does not affect the DSTV at all, and therefore, it cannot be reconstructed from the DSTV (i.e., there does not exist an IS).

Part b): Assume we have already proven part c) of this proposition, and let us denote by $0 \leq k_o(p) < \infty$ the first sample in $\{h(p + kR)\}_{k=0}^\infty$ which is not zero, for $R \geq p \geq 1$. Now for $M-1 \geq n \geq 0$,

$$x_{sR}(n) = x(n) h(R - (n)_R + k_o(R - (n)_R)R);$$

$$s = k_o(R - (n)_R) + 1 + \frac{n - (n)_R}{R}. \quad (\text{A4})$$

Since $h(R - (n)_R + k_o(R - (n)_R)R) \neq 0$, there exists an IS due to (A4) and Proposition 5, part c).

Part c): The necessity of the conditions for IS is trivially verified in the IS definition. The sufficiency is proved using a similar argument to the one done in the proof of Proposition 4, part b). Assume there exists a synthesis system \mathcal{S} such that $\{x(n)\}_{n=0}^{MJ-1} = \mathcal{S}\{x_{sR}\}_{s=1}^{\Delta}$, i.e., from a finite prefix of length ΔI of the DSTV, one reconstructs the samples $\{x(Mr - p)\}_{r=1, p=1}^{J, M}$. Note that $MJ = IR$, and therefore, after subtracting the terms depending on these known first IR samples of $x(n)$ from $\{x_{sR}\}_{s=I+1}^\infty$, we obtain $\{x(n)\}_{n=IR}^{2IR-1}$ as the output of \mathcal{S} with input $\{x_{sR}\}_{s=I+1}^{I(\Delta+1)}$. Continuing in this manner we have constructed an IS. \square

APPENDIX B

Proof of Proposition 6:

Part a): We assumed that P is an LM with respect to $h(n)$ [using $\hat{h}(n)$]. Therefore, for every causal se-

quence $x(n)$, there exists another causal sequence $y(n)$, such that for $M-1 \geq l \geq 0$ and $\infty > s \geq 1$,

$$\sum_{t=0}^{\infty} \hat{h}(sR - l - Mt) y(l + Mt) = \sum_{m=0}^{M-1} \hat{P}(l, m) \sum_{r=0}^{\infty} h(sR - m - Mr) x(m + Mr). \quad (\text{B1})$$

The matrix \hat{P} is nonsingular, and therefore, for every $M-1 \geq l \geq 0$, there exists $M-1 \geq m_o(l) \geq 0$, such that $\hat{P}(l, m_o(l)) \neq 0$. Equation (B1) holds in particular for the sequence $x^{(l, \rho)}(m + Mr) = \delta(m - m_o(l)) \delta(r - \rho) [1/\hat{P}(l, m_o(l))]$, i.e., there exist sequences $y^{(l, \rho)}(n)$ for $M-1 \geq l \geq 0$, $(J-1) \geq \rho \geq 0$, such that

$$\sum_{t=0}^{\infty} \hat{h}(sR - l - Mt) y^{(l, \rho)}(l + Mt) = h(sR - m_o(l) - M\rho) \quad (J-1) \geq \rho \geq 0. \quad (\text{B2})$$

Consider (B2) for $l \geq s \geq 1$. This equation is rewritten as

$$\hat{H}_l y^{(l, \rho)} = h_{m_o(l)}^{(\rho)}; \quad (J-1) \geq \rho \geq 0 \quad (\text{B3})$$

where the s th entry of the vector $h_{m_o(l)}^{(\rho)} \in \mathbb{C}^I$ is $[h(sR - m_o(l) - M\rho)]$, the t th entry of the vector $y^{(l, \rho)} \in \mathbb{C}^J$ is $y^{(l, \rho)}(l + Mt)$, and the (s, t) element of the $I \times J$ -dimensional matrix \hat{H}_l is $h(sR - l - Mt)$. Since we assumed that there exists an FTIS for $h(n)$, one can in particular reconstruct $\{x(m_o(l) + M\rho)\}_{\rho=0}^{J-1}$ from $\{x_{sR}(m_o(l))\}_{s=1}^I$, and therefore, the J vectors $h_{m_o(l)}^{(0)}, \dots, h_{m_o(l)}^{(J-1)}$ in \mathbb{C}^I are linearly independent for every value of $m_o(l)$. Thus, due to (B3), $\text{Range}(\hat{H}_l)$ contains a linear subspace of dimension J . Since \hat{H}_l is an $I \times J$ -dimensional matrix, it follows that the columns of \hat{H}_l are linearly independent. Since this argument holds for every $M-1 \geq l \geq 0$, when $\hat{h}(n)$ is the analysis filter, the samples $\{y(Mr - p)\}_{r=1, p=1}^{J, M}$ can be reconstructed from $\{y_{sR}\}_{s=1}^I$. Due to Proposition 5, part c), this guarantees the existence of an IS. Furthermore, by following the construction of the IS given there, we notice that in our case (i.e., $\Delta = 1$), this is essentially an FTIS for $\hat{h}(n)$.

A Corollary from the Above Proof: If P is an LM with respect to $h(n)$, then $\text{Range}(\hat{H}_l) \supseteq \text{Range}(H_m)$ for every (l, m) such that $\hat{P}(l, m) \neq 0$.

Part b): Since P is an LM, there exists a causal sequence $y(n)$ for which $Y_{sR} = \hat{X}_{sR}$, $\infty > s \geq 1$. From part a) of this proposition, there exists an FTIS for $\hat{h}(n)$ which reconstructs $y(\Delta)$ from $\{Y_{sR}\}_{s=1}^\infty$, i.e., from $\{\hat{X}_{sR}\}_{s=1}^\infty$. Therefore, $y(n)$ in Fig. 2 is exactly the causal sequence for which (5) holds. Due to (5), in Fig. 2,

$$\hat{Y}_{sR} = P^{-1} Y_{sR} = P^{-1} \hat{X}_{sR} = X_{sR}; \quad \infty > s \geq 1, \quad (\text{B4})$$

and thus, $x(n)$ is reconstructed from $\{\hat{Y}_{sR}\}_{s=1}^{\infty}$ by the FTIS for $h(n)$, which proves our claim.

Proposition 7: This is an immediate consequence of the following lemma.

Lemma 1: P is an LM with respect to $h(n)$, iff, for the MJ sequences $x^{(m_o, \rho)}(m + Mr) = \delta(m - m_o) \delta(r - \rho)$, $J - 1 \geq \rho \geq 0$, $M - 1 \geq m_o \geq 0$, there exist corresponding MJ sequences $y^{(m_o, \rho)}(n)$, for which (B1) holds with every nonzero element of \hat{P} replaced by 1.

Proof of the Lemma:

a) *Necessity of the Above Condition:* It follows immediately from the definition of the LM, when using the input sequences $x^{(m_o, \rho)}(n) [1/P(l, m_o)]$ for every l for which $\hat{P}(l, m_o) \neq 0$.

b) *Sufficiency of the Above Condition:* Assume the MJ sequences $y^{(m_o, \rho)}(n)$ exist, then it is fairly easy to verify that for an arbitrary casual input $x(n)$, (B1) holds for the causal sequence

$$y(l + Mt) \triangleq \sum_{m_o=0}^{M-1} \sum_{\rho=0}^{J-1} \sum_{\theta=0}^{\lfloor t/J \rfloor} \hat{P}(l, m_o) \cdot x(m_o + M\rho + \theta JM) \cdot [y^{(m_o, \rho)}(l + M(t - \theta J))]. \quad (\text{B5})$$

Proof of Proposition 8: For P which is an LM with respect to $h(n)$, both Range (\hat{H}_l) and Range (H_m) are linear subspaces of \mathbb{C}^I of dimension J , for every $[M - 1 \geq l, m \geq 0]$ [since both $h(n)$ and $\hat{h}(n)$ possess an FTIS due to Proposition 6, part a)]. Furthermore, as a corollary of Proposition 6, part a), we have essentially shown that Range (\hat{H}_l) = Range (H_m) for every (l, m) such that $[\hat{P}(l, m) \neq 0]$. It is therefore sufficient to prove that [Range (H_m) \neq Range (H_q)] for every (m, q) such that $[\lfloor m/g \rfloor \neq \lfloor q/g \rfloor]$. Since $h(n) \neq 0$ for $R \geq n \geq 1$ (due to Proposition 3), and $h(n)$ is causal, it is easily verified that each column of the matrices $H_m(H_q)$ possess its first nonzero element in the row $s_o(t) \triangleq (\lceil (tI + v)/J \rceil + 1)$ where $v = \lfloor m/g \rfloor$ ($\lfloor q/g \rfloor$, respectively). Moreover, $1 \leq s_o(t) \leq I$ for every $J - 1 > t \geq 0$, and $s_o(t + 1) \geq s_o(t) + 1$, since $I \geq J$. Therefore, if there exists a column t_o for which $s_o(t_o)$ in H_q and H_m differ one from the other, then Range (H_m) \neq Range (H_q). Without loss of generality we assume that $\lfloor m/g \rfloor < \lfloor q/g \rfloor$, and choose $J - 1 \geq t_o \geq 0$ such that $(-t_o I) \pmod{J} = \lfloor m/g \rfloor$ (this is possible since $\gcd(I, J) = 1$). For this value of t_o , it is easily verified that $s_o(t_o)_{H_q} \geq s_o(t_o)_{H_m} + 1$, which completes the proof.

Corollary from the Proof Above: For P which is an LM with respect to $h(n)$, every upper-left submatrix of $\hat{H}_l(H_m)$ whose dimensions are $s_o(\epsilon - 1) \times \epsilon$ has linearly independent columns.

Proof of Proposition 9: Since we assumed that $h(n)$ possesses an FTIS, and that $R = M$, it follows from Propositions 2 and 3 that TDIS exists for $h(n)$. Therefore, for every causal filter $h(n)$, when using $\hat{h}(n) = h(n)$, and the sequence $y(n)$ generated by this TDIS from

$\{\hat{X}_{sR}\}_{s=1}^{\infty}$, (5) holds independently of the specific matrix P , which completes the proof.

Proof of Proposition 10: We construct here the requested FIR filter as follows. The filter is of length R , and satisfies $\hat{h}(1) = \dots = \hat{h}(g)$, $\hat{h}(g + 1) = \dots = \hat{h}(2g)$, etc. Therefore, for every $M - 1 \geq (l, m) \geq 0$ such that $\lfloor l/g \rfloor = \lfloor m/g \rfloor$ and for every $t \geq 0$, $s \geq 1$, $\hat{h}(sR - l - Mt) = \hat{h}(sR - m - Mt) = h(sR - m - Mt)$. Therefore, for every block-diagonal matrix \hat{P} with $g \times g$ -dimensional blocks, (B1) with $h(n)$ and $\hat{h}(n)$ holds for every causal sequence $x(n)$, where $y(n)$ is given by

$$y(l + Mt) \triangleq \sum_{m=g \lfloor l/g \rfloor}^{g \lfloor l/g \rfloor + (g-1)} \hat{P}(l, m) x(m + Mt). \quad (\text{B6})$$

Proof of Proposition 11:

Part a): Let \hat{P} be given by (6). For every filter $h(n)$, with an FTIS, choose $\hat{h}(gv - u) \triangleq h(gv - \pi(u))$ for $g - 1 \geq u \geq 0$, $v \geq 1$, and then for every causal sequence $x(n)$, (B1) holds for $y(n)$ which is given by

$$y(u + gv) \triangleq x(\pi(u) + gv) \lambda(u + g(v)); \quad g - 1 \geq u \geq 0, \quad v \geq 0. \quad (\text{B7})$$

Therefore, such a \hat{P} is a ULM.

Part b): We shall construct an FIR filter with the desired property, as follows. $h(1) = \dots = h(R) \neq 0$ and $h(R + 1), \dots, h(2R)$ which have R different values. First, we shall show that for this $h(n)$, Range (H_m) is a different subspace of \mathbb{C}^I for at least each one of some g consecutive values of $M - 1 > m \geq 0$. Now, $s_o(J - 1) - s_o(0) \geq J$ if $\lceil ((J - 1)(I - J) + v)/J \rceil \geq \lceil v/J \rceil + 1$, where $v = \lfloor m/g \rfloor$. This holds for $I > J > 1$ and $(v)_J = 0$, and then there exists a $(J - 1) \geq t_o \geq 0$ with $s_o(t_o + 1) \geq s_o(t_o) + 2$. Since $H_m(s_o(t_o), t_o)$ does not depend on m , whereas $H_m(s_o(t_o) + 1, t_o)$ has g different values for $vg + (g - 1) \geq m \geq vg$, it follows that for each of these g values of m , Range (H_m) is a different subspace of \mathbb{C}^I . For $I > J = 1$, H_m is a vector in \mathbb{C}^I , and this result is immediate for $v = 0$ from the structure of $h(n)$.

Due to the corollary of Proposition 6, and Proposition 8, at least for $g - 1 \geq l \geq 0$, the l th row of an LM \hat{P} has exactly one nonzero element.

The proof is completed using the following lemma.

Lemma 2: If the matrix \hat{P} , which has a nonzero (l, m) th element, is an LM with respect to $h(n)$, using some analysis filter $\hat{h}(n)$, then the matrix $\hat{P}^{(+)}$ which has in addition nonzero $(l + gv, m + gv)$ th elements is also an LM with respect to $h(n)$, using the same $\hat{h}(n)$.

Proof of Lemma 2: Consider the characterization of LM in Lemma 1. For $(m_o \neq m)_g$, the sequences $y^{(m_o, \rho)}(n)$ of $\hat{P}^{(+)}$ coincide with those of \hat{P} . Since $\hat{P}(l, m) \neq 0$, we know (again due to Lemma 1) that there exist sequences $y^{(m, \rho)}(n)$, $J - 1 \geq \rho \geq 0$, which satisfy

$$\sum_{t=0}^{\infty} \hat{h}(sR - l - Mt) y^{(m, \rho)}(l + Mt) = h(sR - m - M\rho); \quad \infty > s \geq 1. \quad (\text{B8})$$

And we only have to derive from them the sequences $y^{(m+gv,\rho)}(n)$ satisfying

$$\begin{aligned} & \sum_{t=0}^{\infty} \hat{h}(sR - l - gv - Mt) y^{(m+gv,\rho)}(l + gv + Mt) \\ & = h(sR - m - gv - M\rho); \quad \infty > s \geq 1. \end{aligned} \quad (\text{B9})$$

For every ρ, v there exists $(J - 1) \geq \rho' \geq 0$, such that $(\rho'I - \rho I - v)_J = 0$ (since $\gcd(J, I) = 1$), and we denote by $\epsilon \triangleq (\rho' - \rho)$. Now, the solution of (B9) is

$$y^{(m+gv,\rho)}(l + gv + Mt) \triangleq \begin{cases} y^{(m,\rho')}(l + Mt + M\epsilon) & t \geq \max(-\epsilon, 0) \\ 0 & \text{otherwise} \end{cases}.$$

This completes the proof of the lemma, provided that we show that for $\epsilon > 0$, $y^{(m,\rho')}(l + Mt) = 0$ for $0 \leq t \leq (\epsilon - 1)$. The right-hand side of (B8) is zero for ρ' and $s \leq s_o(\epsilon - 1)$ (since $s_o(\epsilon - 1)R - m - M\rho' = s_o(\epsilon - 1)R - m - M(\epsilon - 1) - M(\rho + 1) \leq R - M \leq 0$), and therefore, (B8) contains a subsystem of $s_o(\epsilon - 1)$ equations, in the ϵ unknowns $\{y^{(m,\rho')}(l + Mt)\}_{t=0}^{\epsilon-1}$, with coefficients matrix which is an upper-left submatrix of \hat{H}_t of dimensions $s_o(\epsilon - 1) \times \epsilon$ and zero, free vector. Thus, the proof is completed due to the corollary of Proposition 8.

Completion of the Proof of Proposition 11: Denote the only nonzero element in the l th ($g - 1 > l \geq 0$) row of an LM \hat{P} with respect to $h(n)$ as the $(l, \pi_o(l))$ th element. Assume that there exists an LM \hat{P} with respect to $h(n)$ with $\hat{P}(l + gv_o, m + gv_o) \neq 0$ and $m \neq \pi_o(l)$. Then, with the same analysis filter $\hat{h}(n)$, $\hat{P}^{(+)}$ is also an LM with respect to $h(n)$ due to Lemma 2. However, the l th row of $\hat{P}^{(+)}$ has two nonzero elements, and contradiction arises.

APPENDIX C

We assume throughout that the given MDSTT $\{\hat{X}_{sR}\}_{s=1}^{\infty}$ is in l_1 , i.e., $\sum_{s=1}^{\infty} \|\hat{X}_{sR}\|_1 < \infty$. Using the Cauchy-Schwartz inequality, $\sum_{s=1}^{\infty} \|\hat{X}_{sR}\|_2 < \infty$, and therefore, $\sum_{s=1}^{\infty} \|\hat{x}_{sR}\|_2 < \infty$ (since $\|A\mathbf{v}\|_2 \leq [\lambda_{\max}(A * A)]^{1/2} \|\mathbf{v}\|_2$ [14]). This implies in particular that $\sum_{s=1}^{\infty} |\hat{x}_{sR}(M - p)| < \infty$ for every $M \geq p \geq 1$, and therefore,

$$\begin{aligned} & \sum_{k=1}^{\infty} |u^{(p)}(k)| \\ & \leq \sum_{s=1}^{\infty} |\hat{x}_{sR}(M - p)| \sum_{k=1}^{\infty} |h^*(sR + p - kM)| < \infty \end{aligned}$$

by applying the triangle inequality to (11b), and the assumption that $h(n)$ is an FIR filter.

Proof of Proposition 12:

a) We define the sequence of matrices $\{Y_k^{(p)}\}_{k=-\infty}^{\infty}$ by

$$Y_k^{(p)} \triangleq \int_{-0.5}^{0.5} Y(f) e^{j2\pi fk} df; \quad -\infty < k < \infty \quad (\text{C1a})$$

$$Y(f) \triangleq B(f)^{-1}, \quad B(f) \triangleq \sum_{k=-(K_h-1)}^{(K_h-1)} B_{-k}^{(p)} e^{-j2\pi fk}; \quad -0.5 \leq f \leq 0.5. \quad (\text{C1b})$$

Each element of the matrix $B(f)$ is a trigonometric polynomial, and due to (16), $B(f)$ is nonsingular for every $+0.5 \geq f \geq -0.5$. Thus, each entry of $Y(f)$ is a well-defined continuous function in $[-0.5, 0.5]$, and the matrices $Y_k^{(p)}$ are well defined.

b) Using the definition of $y_n^{(p)}$ in (19)

$$\begin{aligned} \|y_n^{(p)}\|_2 & \leq \sum_{k=-\infty}^{\infty} \|Y_k^{(p)}\|_2 \|u_{n-k}^{(p)}\|_2 \\ & \leq \left(\sup_k \|Y_k^{(p)}\|_2 \right) \sum_{k=-\infty}^{\infty} \|u_n^{(p)}\|_2. \end{aligned}$$

Since each entry of $Y(f)$ is in $L_1[-0.5, 0.5]$, it follows from the Riemann-Lebesgue lemma [15] that both $\lim_{|k| \rightarrow \infty} \|Y_k^{(p)}\|_{\infty} = 0$ and $\lim_{|k| \rightarrow \infty} \|Y_k^{(p)}\|_1 = 0$, and therefore, also $\lim_{|k| \rightarrow \infty} \|Y_k^{(p)}\|_2 = 0$ [14]. Thus, $\sup_k \|Y_k^{(p)}\|_2 \leq \infty$, and since $\infty > \sum_{k=1}^{\infty} |u^{(p)}(k)| = \sum_{n=1}^{\infty} \|u_n^{(p)}\|_1 \geq \sum_{n=1}^{\infty} \|u_n^{(p)}\|_2$, it follows that $\|y_n^{(p)}\|_2 < \infty$ and $y_n^{(p)}$ is well defined by (19).

c) For $n \geq K_h$, using (19) and (C1), we obtain

$$\begin{aligned} & \sum_{k=-(K_h-1)}^{(K_h-1)} B_{-k}^{(p)} y_{(n-k)}^{(p)} \\ & = \sum_{k=-(K_h-1)}^{(K_h-1)} B_{-k}^{(p)} \sum_{l=-\infty}^{\infty} \int_{-0.5}^{0.5} Y(f) u_{(n-l)}^{(p)} e^{j2\pi f(l-k)} df \\ & = \sum_{l=-\infty}^{\infty} \int_{-0.5}^{0.5} \left(\sum_{k=-(K_h-1)}^{(K_h-1)} B_{-k}^{(p)} e^{-j2\pi fk} \right) \cdot Y(f) u_{(n-l)}^{(p)} e^{j2\pi fl} df \\ & = \sum_{l=-\infty}^{\infty} u_{(n-l)}^{(p)} \int_{-0.5}^{0.5} e^{j2\pi fl} df = u_n^{(p)} \quad (\text{C2}) \end{aligned}$$

where the change of order of the summations over k and l is permitted since one of them is finite, and the other converges as shown above.

Thus, the solution in (19) satisfies (18) at least for $n \geq K_h$.

d) For $u_n^{(p)} = \mathbf{0}$ except for $1 \leq n \leq n_o$, $y_n^{(p)} = \sum_{k=(n-n_o)}^{(n-1)} Y_k^{(p)} u_{(n-k)}^{(p)}$, and therefore,

$$\begin{aligned} \|y_n^{(p)}\|_2 &\leq \sum_{k=(n-n_0)}^{(n-1)} \|Y_k^{(p)}\|_2 \|u_{(n-k)}^{(p)}\|_2 \\ &\leq \left(\max_{(n-n_0) \leq k \leq (n-1)} \|Y_k^{(p)}\|_2 \right) \left(\sum_{k=1}^{n_0} \|u_k^{(p)}\|_2 \right). \end{aligned}$$

However, we have already shown that $\lim_{|k| \rightarrow \infty} \|Y_k^{(p)}\|_2 = 0$, and therefore, $\lim_{n \rightarrow \infty} \|y_n^{(p)}\|_2 = 0$, i.e., the solution of (19) is a stable solution.

e) The zeros of $\det \{ \sum_{k=-(K_h-1)}^{(K_h-1)} B_{+k}^{(p)} z^{-k} \}$ are organized in pairs (z_1, z_2) such that $z_1 \bar{z}_2 = 1$. The different solutions of (18) (for $n \geq K_h$) correspond to different partitions of these $(2K_h - 1)J$ zeros between the causal and anticausal components of the sequence $\{Y_k^{(p)}\}_{k=-\infty}^{\infty}$, and there exists *only one partition* which leads to a stable solution [16].

Proof of Proposition 13:

Part a): It is sufficient to prove that

$$(\forall \epsilon > 0)(\exists T_\epsilon)(\forall n \geq 1) \left(\left\| \sum_{|k| \geq T_\epsilon} Y_k^{(p)} u_{(n-k)}^{(p)} \right\|_2 < \epsilon \right). \quad (C3)$$

However, as shown above,

$$\begin{aligned} &\left\| \sum_{|k| \geq T_\epsilon} Y_k^{(p)} u_{(n-k)}^{(p)} \right\|_2 \\ &\leq \left(\sum_{k=1}^{\infty} \|u_n^{(p)}\|_2 \right) \left(\sup_{|k| \geq T_\epsilon} \|Y_k^{(p)}\|_2 \right) \end{aligned}$$

and since $\lim_{|k| \rightarrow \infty} \|Y_k^{(p)}\|_2 = 0$, and $\sum_{k=1}^{\infty} \|u_n^{(p)}\|_2 < \infty$, (C3) holds. We note here that T_ϵ depends only on the l_1 norm of the given MDSTT, and so is Q_ϵ . Thus, if the input MDSTT's are uniformly l_1 bounded sequences, then Q_ϵ and T_ϵ can be determined according to this given bound.

Part b): For every $\epsilon > 0$ we find a value of T_ϵ which guarantees a reconstruction error below $\epsilon/2$. We then approximate $\{Y_k^{(p)}\}_{k=-(T_\epsilon-1)}^{(T_\epsilon-1)}$ by

$$\tilde{Y}_k^{(p)} \triangleq \frac{1}{Q} \sum_{m=0}^{(Q-1)} Y\left(\frac{m}{Q}\right) e^{j(2\pi/Q)mk}, \quad |k| \leq (T_\epsilon - 1). \quad (C4)$$

Now

$$\begin{aligned} &\left\| y_n^{(p)} - \sum_{k=-(T_\epsilon-1)}^{(T_\epsilon-1)} \tilde{Y}_k^{(p)} u_{(n-k)}^{(p)} \right\|_2 \\ &\leq \epsilon/2 + \left\| \sum_{k=-(T_\epsilon-1)}^{(T_\epsilon-1)} (Y_k^{(p)} - \tilde{Y}_k^{(p)}) u_{(n-k)}^{(p)} \right\|_2 \end{aligned}$$

$$\begin{aligned} &\leq \epsilon/2 + \left(\max_{|k| \leq (T_\epsilon-1)} \|Y_k^{(p)} - \tilde{Y}_k^{(p)}\|_2 \right) \\ &\cdot \left(\sum_{n=1}^{\infty} \|u_n^{(p)}\|_2 \right). \quad (C5) \end{aligned}$$

Since $\sum_{n=1}^{\infty} \|u_n^{(p)}\|_2 < \infty$, and T_ϵ is finite, it is sufficient to show that for every $|k| \leq (T_\epsilon - 1)$, $\lim_{Q \rightarrow \infty} \|Y_k^{(p)} - \tilde{Y}_k^{(p)}\|_2 = 0$. However, every entry of $Y(f) e^{j2\pi f k}$ is continuous function, and is thus Riemann integrable. Therefore, due to (C1) and (C4), every entry of $(Y_k^{(p)} - \tilde{Y}_k^{(p)})$ approaches zero as Q approaches infinity [17], and so is $\|Y_k^{(p)} - \tilde{Y}_k^{(p)}\|_2$ [14].

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