IMAGE PARTITIONING FOR FRACTAL CODING UNDER RATE AND COMPLEXITY CONSTRAINTS

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Abstract

Fractal image coding is a relatively new technique for compact representation of an image by exploiting self-similarities between parts of the image and other parts in it at a higher resolution. The various parts at different resolutions are consequences of a partition grid and splitting criterion applied to the image. In this work, we propose a partitioning criterion which takes in consideration the computational complexity of the encoding process. Based on this criterion we develop an algorithm for adaptive image partitioning achieving a range of rates under a computational complexity constraint. The proposed algorithm results in a reduction of the computational complexity as compared to other known algorithms at the same rate-distortion operating point.

1. Introduction

The first coding algorithm was suggested by Jacquin [1] according to which the image is divided into non-overlapping blocks covering the whole image. A new partition, into larger blocks, is also performed. For each small block a search is done, looking after the best match under a predefined transformation type (including spatial contraction) to a larger (higher resolution) block. The fractal code representing the image is composed of the union of all transformation parameters. Compression is achieved if the amount of information describing these parameters is less than the amount needed to describe the original (at the price of some distortion). The task of finding self-similarities is of high computational complexity and is the major drawback of fractal coding. Decoding is done by iteratively applying the transformations to any initial image. Alternatively, we used a fast hierarchical decoding

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algorithm introduced by Baharav et. al. [2] in order to reduce the computational complexity at the decoding stage.

2. Quadtree Partitioning Criteria

It is well known that coding efficiency can be improved by using adaptive image partitioning methods such as the well-known Quadtree partitioning [3,4]. When a non-uniform partition is used, smooth areas are covered by few large blocks and active areas, containing many details (e.g., texture), are covered by small blocks to better capture the image characteristics. According to the partition criterion presented at [3,4], here denoted as the "*Threshold criterion*", each range-block is divided into four non-overlapping sub-blocks if its collage error (matching distortion) is above a predefined threshold. The *Threshold* criterion doesn't enable a direct control of rate or computational complexity. In this paper, we aim at finding criteria for adaptive partitioning to reduce the reconstruction errors and the computational complexity for a given rate. The optimal but not practical solution is to go over all possible quadtree structures subject to the rate constraint and to choose the one with the minimum distortion.

2.1 A rate-distortion based quadtree partitioning criterion

We examined a *rate-distortion* based partitioning criterion (for abbreviation, denoted here as R-D), which gives priority to partitioning blocks producing the highest reduction in distortion per added bit to the representation of the sub-blocks [5]. This sub-optimal solution results in smaller reconstruction errors than obtained by other reported criteria (see Fig. 1). However, the need to find the matching blocks to the sub-blocks, before a splitting decision could be made, leads to higher computational complexity than needed by the *threshold* criterion.

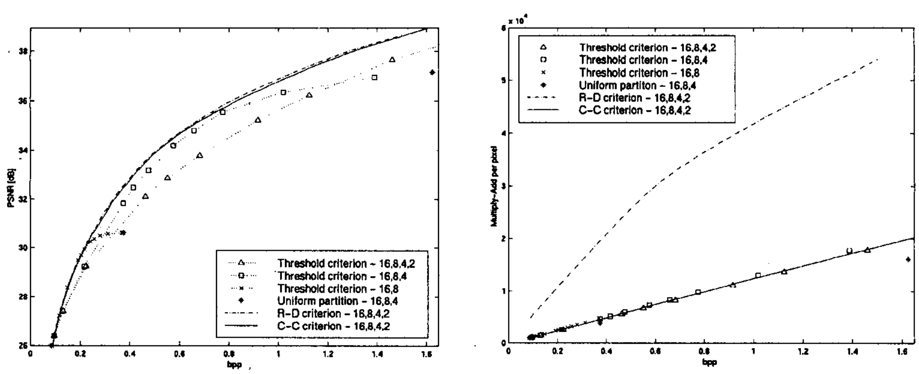
2.2 A partitioning criterion based on computational-complexity

We propose first a quadtree partitioning criterion, which reduces the distortion under a complexity constraint. This new criterion: *Collage-error – Computational-complexity* (denoted here as C-C, for abbreviation) differs from those proposed in other works by taking into account the computational complexity needed while performing the coding process.

The idea is to start with a uniform partition into large blocks. For each block a gain value is computed as the ratio of the block's distortion (collage-error) to the computational complexity needed to find the best transformation for its sub-blocks. A list of blocks ordered in descending gain values is produced. At each stage, the block at the top of the list is partitioned. For each new sub-block created, a new gain value is calculated and placed in a proper location in the descending list. The partitioning process continues until a computational complexity constraint is met. This criterion reduces the computational complexity needed but doesn't provide yet a direct control of the rate – which we consider in section 3.

2.3 Performance comparison

Fig. 1 presents a comparison of reconstruction errors and computational complexity obtained by applying the above mentioned three partitioning criteria to the 256 gray level image "Lena" of size 512x512. It can be seen in Fig. 1(a) that the *threshold* criterion results in higher reconstruction errors than obtained by using either the R-D or the C-C partitioning criteria. In addition, it can be seen that the reconstruction errors obtained by using the C-C criterion are very close to those obtained by applying the R-D criterion. Comparing the computational complexity required by different coding algorithms (Fig. 1(b)) proves that the C-C partitioning criterion is superior to the R-D criterion, but is similar to the *threshold* criterion. That is, it achieves a distortion performance close to that of R-D, but at a much reduced complexity.



(a) (b)

Fig 1: Simulation results for the image "Lena" of size 512x512. The numbers 16,8,4 and 2 indicate the sizes of range blocks used. R-D denotes a rate-distortion based splitting criterion. C-C denotes a collage-error – computational-complexity splitting criterion. (a) Reconstruction errors. (b) Computational complexity

3. An adaptive Partitioning Algorithm with Rate Control

We propose now an adaptive partitioning algorithm which combines the last two criteria (R-D and C-C) and provides rate control. This algorithm uses two top-down passes. After the first pass, a quadtree structure is determined (Fig. 2(a)), which meets a complexity constraint using the descending gain list. The first pass follows the same procedure described for the C-C based partitioning algorithm. This quadtree structure dictates an upper limit to achievable rates. At the end of this top-down pass, each non-terminal node (Fig. 2(a)) contains the gain value (reduction in distortion per bit) which results when the block (represented by the node) is partitioned into sub-blocks. In the second pass, a sub-tree structure (Fig. 2 (b)) is selected from the given quadtree structure by applying the R-D based splitting criterion until a designated rate (less or equal to the rate limit dictated by the given computational consraint value) is achieved. The additional computational-complexity involved in carrying out this pass is negligible, since the gain values are available from the first pass. If a rate higher than the limit dictated by the first pass is desired, the computational-complexity constraint value must be increased.

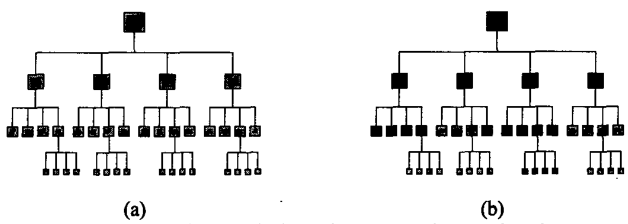


Fig. 2: Adaptive image partitioning under complexity and rate constraints. Each node represents a block in the image. (a) A quadtree structure determined by applying the C-C partitioning criterion until a complexity constraint is achieved. (b) The nodes in black represent a sub-quadtree derived from the former quadtree structure by applying the R-D partitioning criterion until a designated rate is achieved.

4. Experimental results

The above algorithm was applied to the image "Lena" of size 512x512. The reconstruction errors of this algorithm are bounded between those achieved by applying the R-D and C-C based partitions. At the same time, the computational complexity is maintained as low as was obtained when using the C-C criterion. The reconstruction results of this algorithm are not shown here because it is hard to notice visually their difference from the results obtained by the R-D and C-C based partitions (as could be expected from the results shown in Fig. 1).

Three reconstructed images are presented in Fig. 3. The image was coded at the rate of 0.3bpp using range blocks of maximum size of 16x16 and minimum size of 2x2. The reconstructed images shown were coded using the 'Threshold', R-D and, C-C partitioning criteria. It is worth mentioning again that the distortion obtained by using the proposed algorithm in section 3 are bounded by those obtained by using the R-D and C-C partitionings, and therefore are not presented here. As seen in Fig. 3, the reconstructed image related to the 'Threshold' criterion suffers from blockiness effects (at 0.3bpp) as opposed to the other two. Also, it is hard to distinct between the R-D and C-C related images (about 0.18dB difference in PSNR). However, the computational complexity of the R-D partitioning is about four times the complexity of both the C-C and 'Threshold' partitioning approaches.

(a) PSNR 30.38dB (b) PSNR 31.54dB (c) PSNR 31.36dB

Fig. 3: Reconstructed "Lena" images coded at 0.3bpp using range blocks of sizes 16x16 to 2x2 using: (a) 'Threshold' partitioning criterion. (b) R-D partitioning criterion. (c) C-C partitioning criterion.

5. Conclusion

New partitioning criteria were presented allowing consideration of the computational complexity during the encoding process. An adaptive partitioning algorithm was proposed having several benefits. It results in reconstruction errors which are close to those obtained by applying a *rate-distortion* criterion while reducing the computational complexity by a factor of about four. It also enables a direct control of both rate and complexity as opposed to other reported fractal image coding algorithms. Finally, it is simple to implement and allows to combine it with block classification methods [4] to further reduce the computational complexity.

6 References

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