Simple Adaptation of Vector-Quantizers to Combat Channel Errors

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Abstract - *Vector-Quantization* (VQ) is an effective and widely-used method for low-bit-rate communication of speech and image signals. A common assumption in the analysis of VQ systems is that the compressed digital information is transmitted through a perfect channel. Under this assumption, quantizing distortion is the only factor affecting output signal fidelity. However, in physical channels, errors may be present, degrading overall system performance. In order to reduce performance degradation, previous authors suggested to optimally redesign the VQ for noisy channels ("noisy" VQ). The "noisy" VQ results in smaller distortion as compared to the original ("noiseless") VQ for the specific channel it was designed for. The main drawback of this approach is the need to design and store, both in the transmitter and the receiver, several codebooks for different Bit-Error-Rates (BER). In this paper, we show that a simple gain adaptation, which depends on the channel BER, also improves system performance, while using the original ("noiseless") VQ design. The improvement in some cases, as shown by numerical examples, is close to what can be achieved by the optimal approach.

I. Introduction

Vector-Quantization (or *Scalar-Quantization* as a particular case) is a method for mapping signals into digital sequences. In most Signal Processing applications the *source* emits signal samples over an infinite alphabet. These samples should be sent to the *destination* with the highest fidelity possible. The VQ encodes the source output into a digital sequence that is transmitted through the *channel*. The *decoder*'s goal is to reconstruct source samples from this digital sequence. Since analog sources cannot be represented perfectly by digital information, some *Quantization Distortion* must be tolerated. The structure of a VQ-based transmission system is shown in Fig. 1. The scale factor α is introduced here for later reference.

In every channel transmission the VQ encodes a K-dimensional vector of source samples - $\underline{x}(n)$ into a channel symbol $y(n)=\psi\{\underline{x}(n)\}$. A channel symbol y is taken from a finite channel alphabet which, without loss of generality, is represented by the indices $i=0,1,\ldots,N-1$. We assume a memoryless channel, therefore its output $\hat{y}(n)$ is a random mapping of its input, characterized by the probability $q_{ji}=Pr\{\hat{y}(n)=j|y(n)=i\}$.

Finally, the decoder converts the channel symbol into an output reconstruction vector $\hat{\underline{x}}(n)=\phi\{\hat{y}(n)\}$, which is hopefully "close" to the input vector. The set of all reconstruction vectors (or *codevectors*) is the VQ codebook.

We define a *distortion measure* $d(\underline{x},\hat{\underline{x}})$ between the input vector and the reconstruction vector. Throughout, we shall use the Squared-Error distortion measure:

$$d(\underline{x},\hat{\underline{x}})=\|\underline{x}-\hat{\underline{x}}\|^2 \quad (1)$$

The Knowledge of source statistics $p(\underline{x})$ or a representative *training sequence* is assumed. For simplicity we assume that $\underline{x}$ has a zero mean. The performance of the system is measured in terms of the average distortion: $D=E[d(\underline{x},\hat{\underline{x}})]$.

In most VQ applications the channel is assumed to be noiseless [1]-[3], so that no errors occur in transmission. This assumption is based upon using a channel encoder-decoder pair which corrects channel errors.

Upon knowledge of the source statistics, Lloyd's algorithm [2] may be used to design a VQ. The design of a VQ is based in practice on a training sequence. In this case no knowledge of the source statistics is assumed. The design of a VQ can be done using the LBG algorithm [3]. Both methods are iterative and alternatingly apply the *Nearest-Neighbor condition* and the *Centroid condition*. We shall refer to the VQ, designed for a perfect channel as the "noiseless" VQ.

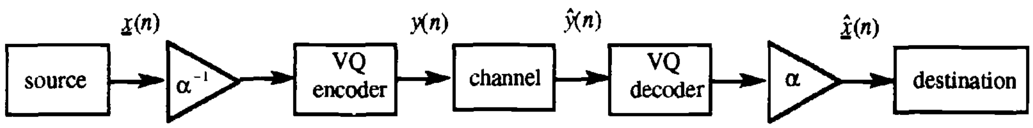

Fig. 1- Vector Quantization system

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II. CHANNEL ERRORS

In the discussion so far we have assumed that the channel is noiseless. In some applications channel-coding is not utilized because of complexity or bit-rate requirements. In such cases the perfect channel assumption may not be justified, and if a channel-error occurs, a wrong codevector is selected [4]-[9].

In the presence of channel errors, and given the transmitted symbol, the received symbol is a random variable. Previous works (e.g., [4,6]) suggest to redesign the VQ by modifying the distortion measure to take all possible output vectors into consideration. This modification results in a *Weighted Nearest-Neighbor condition* and a *Weighted Centroid condition*. These conditions are specific to every channel scenario. Hence, a "noisy" VQ should apply a different partition and a different codebook for each possible channel Bit-Error-Rate (BER). The main drawbacks of this approach are the large memory consumption and the extensive design effort. The system needs to store a codebook *library*, both at the transmitter and the receiver.

In this paper we suggest a different approach for the adaptation of the VQ design to combat channel-errors. The main idea is to scale the "noiseless" VQ codevectors by a factor $0<\alpha\leq 1$. Scaling the VQ codevectors by α is equivalent to applying two gain multipliers in the signal path, as shown in Fig. 1. The purpose of the scaling is to make the distance between any pair of codevectors smaller. Hence, channel errors will cause smaller distortions. On the other hand, the VQ is no longer optimal for the given source, and the Quantization-distortion increases. It is especially clear that the overload error gets larger as the scaling factor α used is smaller. This trade-off results in the existence of an optimal value of α which minimizes the overall average distortion. We shall refer to this method as "α-scaled" VQ.

The first step in designing an "α-scaled" VQ is to design a "noiseless" VQ as usual. Next, the overall distortion, as a function of α is:

(2) $$D(\alpha)=E\{d(\underline{x},\hat{\underline{x}})\}=$$
$$=\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}q_{ji}\int_{S_i(\alpha)}\|\underline{x}-\alpha\phi(j)\|^2\cdot p(\underline{x})\cdot d\underline{x}$$

where $S_i(\alpha)=\left\{\underline{x}:\psi\left(\frac{1}{\alpha}\underline{x}\right)=i\right\}$ is the i-th partition region.

For different channel BERs, $D(\alpha)$ can be optimized off-line over α. For example, due to the optimality of the "noiseless" VQ, for the perfect channel we should get $\alpha_{opt}=1$.

The value of the optimal α becomes smaller as the channel BER increases. For $\alpha\to 0$ all codevectors tend to zero and $D(\alpha)$ approaches the signal variance:

(3) $$D(\alpha\to 0)=\int_{\Omega}\|\underline{x}-\underline{0}\|^2 p(\underline{x})d\underline{x}$$

where Ω is the entire signal space. On the other hand for $\alpha>1$, the distance between every pair of codevectors increases, resulting in larger overall distortion.

In Practice, the transmitter and the receiver need to keep only a single codebook and a Look-Up-Table of optimal scale-factors.

In the following we prove that for a wide class of scalar source PDFs and scalar quantizers, $D(\alpha)$ has a global minimum. The source PDF is assumed to have the following properties:

i. Symmetric and Unimodal, i.e., $p(x)=p(-x)$ and $p(x)\leq p(y)$ for $|x|\geq|y|$.

ii. Log-Concave, i.e., $\partial^2/\partial x^2\left[\log p(x)\right]<0$.

These properties hold for the Uniform, Laplacian, and Gaussian sources [10].

For the scalar quantizer, we use a change of variables $x=\alpha z$ to get:

(4) $$D(\alpha)=E\{d(x,\hat{x})\}=$$
$$=\alpha^3\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}q_{ji}\int_{S_i'}(z-\phi(j))^2 p(\alpha z)\cdot dz$$

where $S_i'=\{z:\psi(z)=i\}$ is the i-th partition interval.

The derivative of $D(\alpha)$ is:

(5) $$D'(\alpha)=$$
$$=\alpha^2\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}q_{ji}\int_{S_i'}(z-\phi(j))^2\cdot\left[3p(\alpha z)+\alpha z p'(\alpha z)\right]\cdot dz$$

Since α^2 is strictly positive for $0<\alpha\leq 1$, $D'(\alpha)=0$ if

(6) $$3p(\alpha z)+\alpha z p'(\alpha z)=0$$

or

$$3+z\frac{\alpha\cdot p'(\alpha z)}{p(\alpha z)}=0$$

Next, we show that for positive values of z, (6) has only one solution. Using the symmetric property of the PDF it is possible to state similar arguments for negative values of z.

Using the log-concave property we state that $p'(\alpha z)/p(\alpha z)$ is a decreasing function of α. The unimodality assures $p'(\alpha z)$ to be negative. Therefore, $\alpha\cdot p'(\alpha z)/p(\alpha z)$ is a negative monotonically decreasing function of α, since both α and $p'(\alpha z)/p(\alpha z)$ has an increasing absolute value. QED.

It should be mentioned that the log-concave property is a sufficient condition for obtaining a single minimum. In specific examples, which where tested numerically, a single minimum was also obtained for non-log-concave PDFs, such as the Gamma source.

III. NUMERICAL EXAMPLES

In order to calculate the optimal scaling factor for a specific system one should calculate $D(\alpha)$ and perform a one-dimensional optimum search. This may be done by standard Non-Linear programming techniques.

For example we analyzed a 4-bit Max-Lloyd scalar quantizer [10] designed for the Laplacian source. The digital information is transmitted over the Binary Symmetric Channel (BSC). In

Fig. 2 the overall distortion is plotted as a function of the scaling factor α, for different values of the BER q. The overall distortion is normalized to the distortion in the case of a perfect channel (quantization distortion only).

It can be seen from Fig. 2 that a considerable improvement is obtained by the scaling method. For example, in the case of 1% BER ($q = 10^{-2}$) a 3-dB gain is attained by using the optimal $\alpha \approx 0.56$ to improve the "noiseless" quantizer ($\alpha = 1$) performance. Moreover, the distortion graph is quite flat about the minimum. This property allows the system to use the same scale factor for a wide ranges of BERs, without reducing much the improvement in performance.

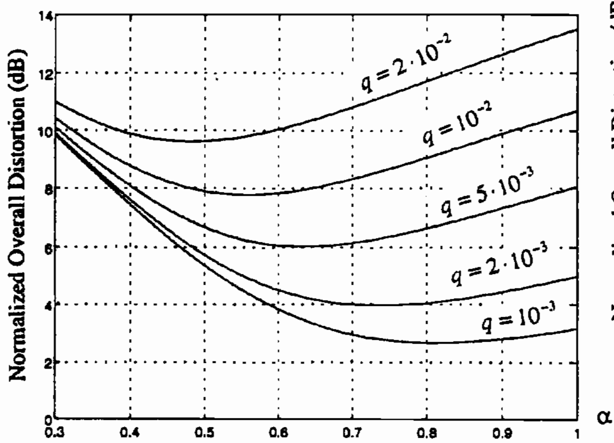

Fig. 2 - The overall distortion of an "α-scaled" 4 bit scalar quantizer, Laplacian source, BSC

In Fig. 3 we plotted the value of the optimal α as a function of the channel BER.

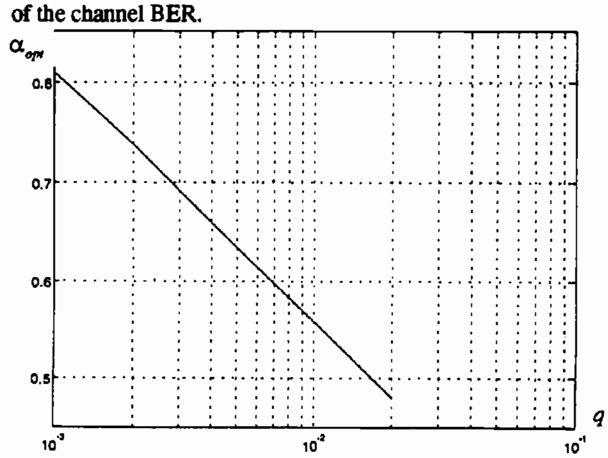

Fig. 3 - The optimal value of the scale factor of an "αscaled" as a function of BER, 4 bit Scalar Quantizer, Laplacian source, BSC

The optimal scale factor values, from Fig. 3 may be stored in a Look-Up-Table, at the receiver and the transmitter.

It is interesting to compare the performance improvement of the "α-scaled" quantizer with the performance improvement of an optimal redesign [4,6]. In Fig. 4, the "α-scaled" quantizer is compared with an optimal redesign as well as with "noiseless" quantizer. As in Fig. 2, distortion is normalized to the quantization distortion of a "noiseless" quantizer under perfect channel conditions.

Fig. 4 demonstrates that in this case the "α-scaled" quantizer approach achieves most of the gain possible by a full quantizer redesign.

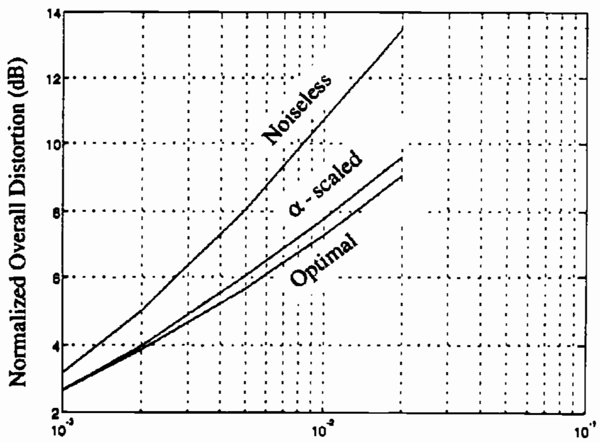

Fig. 4 - An "α-scaled" 4 bit scalar quantizer,compared with an optimal redesign (Normalized distortion, Laplacian source, BSC)

Next, we examine the generalized Gaussian parametric family of distributions [11]. A parameter r controls the shape of the distribution. A small r results in a concentrated PDF, while larger values of r yield flatter distributions. In Fig. 5 the gain in performance of an "α-scaled" quantizer relative to the "noiseless" quantizer ($\alpha = 1$) is compared for several values of r.

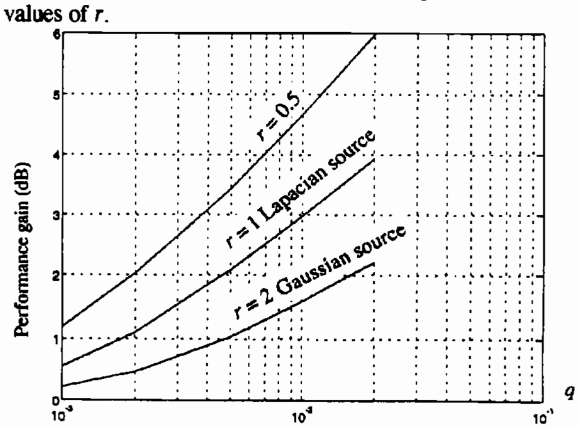

Fig. 5 - Performance Gain of an "α-scaled" 4 bit Scalar Quantizer as a function of BER (Generalized Gaussian source with parameter r, BSC)

We can see that the scaling of a quantizer is more efficient in combating channel errors for concentrated distributions. The reason for this is that scaling by $0 < r < 1$ reduces the quantization levels thus increasing the overload quantization error, particularly for wide distributions. Therefore, narrow distributions are better suited for the proposed adaptation method.

The last example presented in Fig. 6 and Fig. 7 is based upon an 8 bit, 1 bit/sample VQ. The VQ was designed for a Markov Gaussian source with correlation coefficient $\rho = 0.9$. The performance gain as a function of the BER, due to the use of a scale factor, is shown Fig. 6, while the optimal scale factor, α_{opt}, is drawn in Fig. 7.

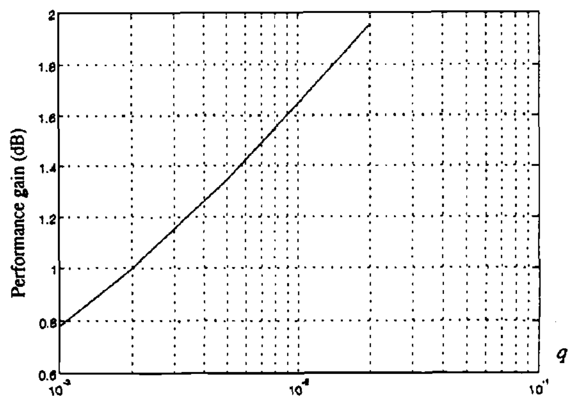

Fig. 6 - Performance Gain of an "α-scaled" 8 bit, 1bit/sample Vector Quantizer as a function of BER, (Gaussian source, BSC)

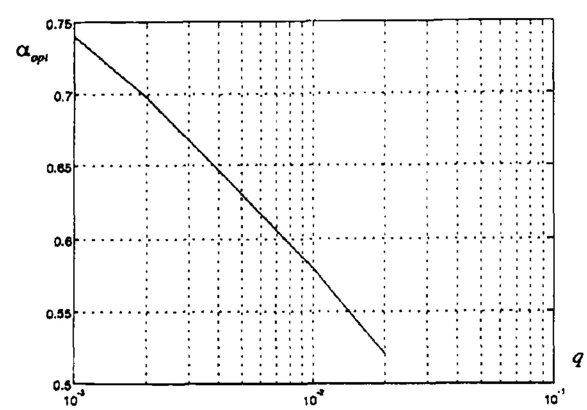

Fig. 7 - Optimal scaling factor of an "α-scaled" 8 bit, 1bit/sample Vector Quantizer as a function of BER, (Gaussian source, BSC)

The application of the scaling factor derived from Fig. 7 is done in the same way as for a scalar quantizer. It should be mentioned that the value of α_{opt} is applicable only for the specific quantizer it was designed for. Other VQs constructed for the same source will have other codevectors. Even for the same codevectors different index-assignments [6] may be chosen resulting in possibly different scale factor values.

IV. Conclusions

In this paper we suggested a simple adaptation method of Scalar and Vector Quantizers to combat channel error effect. We showed that a simple scale factor reduces the overall system distortion. We present numerical examples which demonstrate the performance gain over the corresponding "noiseless" quantizers. For example, in the case of a 4-bit Max-Lloyd scalar quantizer, designed for the Laplacian source, binary symmetric channel (BSC) with 1% error rate, the proposed "α-scaled" quantizer presents a 3-dB performance gain over the "noiseless" quantizer. The optimal redesign approach presents a 3.5-dB gain, while demanding a much higher memory budget and an extensive design procedure.

REFERENCES

[1] Anderson J.B. and Mohan S.: *Source and Channel Coding, An Algorithmic Approach*, Kluwer Academic Publishers, Boston 1991.

[2] Gersho A. and Gray R.M. , *Vector Quantization and Signal Compression*, Kluwer Academic Publishers, Boston 1991.

[3] Y. Linde, A. Buzo and R.M. Gray :"*An algorithm for Vector Quantization Design*", IEEE Tran. Comm., Jan. 1980, pp. 84-95.

[4] Rydbeck N., Sundberg C-R W. : "*Analysis of Digital Errors in Nonlinear PCM Systems*" IEEE Trans. Comm. Jan. 1976 pp. 59-65.

[5] Zelinski R. "*Effects of Transmission Errors on the Mean-Squared Error Performance of Transform Coding Systems*" IEEE Trans. ASSP, Vol. 27, No. 5, Oct 1979, pp. 531-537.

[6] Farvardin N. : "*A Study of Vector Quantization for Noisy Channels*" , IEEE Trans. Inform. Theory, Vol. 36, No. 4, Jul. 1990, pp. 799-809.

[7] Farvardin N. and Vaishampayan V. : "*Optimal Quantizer Design for Noisy Channels: An Approach to Combined Source-Channel Coding*" IEEE Trans. Inform. Theory,Nov. 1987 ,pp. 155-159.

[8] Farvardin N. and Vaishampayan V. : "*On The performance and Complexilty of Channel Optimized Vector Quantizers*" IEEE Trans. Inform. Theory, Jan. 1991, pp. 155-159.

[9] Kurtenbach A.J. and Wintz P.A. : "*Quantizing for Noisy Channels*" IEEE Trans. Inform. Tech., Apr. 1969, pp. 291-302.

[10] Jayant N.S. and Noll P.: *Digital Coding of Waveforms*, Prentice-Hall, London 1984.

[11] Clarke R.J.: *Transform coding of Images*, Academic Press, London 1985