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DESIGN OF UNIFORM DFT FILTER BANKS OPTIMIZED FOR SUB-BAND CODING OF SPEECH

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ABSTRACT

A new approach for designing uniform DFT analysis/synthesis filter-banks, optimized for sub-band coding (SBC) of speech, is presented. The design is performed by an iterative algorithm, which consists of solving two sets of linear equations in each iteration, aiming to minimize a spectral-domain distortion function. This function takes into account the desired characteristics of analysis/synthesis filter-banks with decimation, quantization, and interpolation, used in SBC. Using the new design approach a 16 Kbps SBC is simulated and is found to achieve similar subjective and objective (SNR) performance, as that of a conventional QMF-based SBC, with only about 40% of the computations.

1. INTRODUCTION

Sub-band coding is a well known method for digital speech coding at medium rates (e.g. 16 Kbits/sec). In a sub-band coder (SBC) the signal is divided into separate bands (typically eight) by using an analysis filter bank. Usually, each band-signal is quantized by a gain-adaptive scalar quantizer. The speech is reconstructed from the quantized band-signals using a synthesis filter bank. A coder of this type is considered to be of "medium complexity", with its most complex part being the filter bank [1].

Two common types of filter-banks are the Quadrature Mirror Filter (QMF) bank [2], and the uniform DFT filter bank [2]. The QMF bank is designed to completely cancel the aliasing due to the decimation of the band-signals (in the absence of quantization), and it is widely used in sub-band speech coders. The QMF-based SBC obtains good quality at medium bit rates. Its drawback, however, is its relatively high implementation complexity.

On the other hand, the DFT bank can be implemented efficiently using the Weighted Overlap Add (WOLA) scheme [2], in which case it is of much lower complexity than the QMF bank, for similar band separation. However, the performance (in terms of subjective quality) of DFT-based SBC, obtained by known filter design methods, was found to be much lower. In a recent work [4], a new design approach was presented, but was not found to sufficiently improve the subjective quality.

In this work we present a method for designing filters for the uniform DFT filter-banks which are optimized for sub-band coding of speech. This is achieved by minimizing a spectral-domain distortion function which takes into consideration the following desired characteristics of an analysis/synthesis filter-bank:

a. Good band-separation in the analysis stage, in order to enhance the redundancy removal by providing uncorrelated band-signals and thereby allowing the design of quantizers which are better matched to the non-stationary properties of the band-signals (in particular, adaptation to the gain in each band).

b. Good band-separation in the synthesis stage, for exploiting the auditory masking effect [3]. The quantization noise in each frequency band is "masked" by a stronger band-signal in the same band, i.e. reducing the loudness at which the noise is perceived. It is important to minimize the leakage of quantization noise from one frequency band to other bands (including adjacent bands if their bandwidth is not narrow [3]), since its masking in other bands can not be controlled, as it depends on signal intensity in those bands.

c. Minimizing aliasing effects (due to the decimation of the band-signals) in the reconstructed speech signal, and providing a close approximation to a unity system, i.e. the overall analysis/synthesis transfer function (in the absence of quantization) should be close to a pure delay.

The paper is organized as follows: section 2 presents the distortion function to be minimized; section 3 presents the iterative design algorithm used; section 4 presents a design example, and section 5 the implementation of a 16 Kbps SBC using the optimized filter banks. Section 6 presents simulation results with new SBC system and, for comparison, also with a convential QMF-based SBC.

2. DISTORTION FUNCTION

The basic equivalent structure of a (complex) uniform DFT filter-bank is shown in Fig. 1. In the analysis stage, the input signal $x(n)$ is demodulated by $\exp(-j\omega_k n)$, filtered by the FIR analysis lowpass filter $h(n)$, and decimated by R:1 to produce the M band signals $X_k(m)$, $k=0,1,\ldots,M-1$. Ideally, these M complex signals, have a bandwidth of $2\pi/M$. In the synthesis stage, the band signals are interpolated by the synthesis lowpass filters $f(n)$, modulated by $\exp(j\omega_k n)$, and summed up to produce the output signal $\hat{x}(n)$.

Using well known z-transform expressions for decimated and interpolated signals, the z-transform of the output signal, when no quantization is applied, is given by:

$$\hat{X}(z) = \frac{1}{R}\sum_{k=0}^{R-1} X(zW_R^k)\frac{1}{M}\sum_{l=0}^{M-1} F(zW_M^{-l})H(zW_R^k W_M^{-l})$$
$$W_M \triangleq e^{j2\pi/M}, \quad W_R \triangleq e^{j2\pi/R} \tag{1}$$

The terms which include $X(zW_R^k)$, for $k \neq 0$, are the aliasing components in the output signal. Hence, we define the aliasing distortion as:

$$E_{al} \triangleq \frac{1}{M^2}\sum_{k=1}^{R-1}\frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{1}{R}\sum_{l=0}^{M-1} F(e^{j\omega}W_M^{-l})H(e^{j\omega}W_R^k W_M^{-l})\right|^2 d\omega \tag{2}$$

The mean squared error (MSE) relative to unity transmission, i.e. pure delay of n_o samples is given by:

$$E_{tr} \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|e^{-j\omega n_o} - \frac{1}{MR}\sum_{l=0}^{M-1} F(e^{j\omega}W_M^{-l})H(e^{j\omega}W_M^{-l})\right|^2 d\omega \tag{3}$$

As shown in Fig. 2, the ideal frequency responses of the filters, are:

$$H_{ideal}(e^{j\omega}) = \begin{cases} M, & 0 < |\omega| < \pi/M \\ 0, & \pi/M < |\omega| < \pi \end{cases}$$
$$F_{ideal}(e^{j\omega}) = \begin{cases} M, & 0 < |\omega| < \pi/M \\ 0, & 2\pi k/R - \pi/M < |\omega| < 2\pi k/R + \pi/M \\ \phi, & 2\pi(k-1)/R + \pi/M < |\omega| < 2\pi k/R - \pi/M \end{cases} \tag{4}$$
$$k = 1,2,\ldots,R/2$$

where ϕ denotes "don't care" regions. The energy of the side-lobes (stop band) of the analysis and synthesis filters is:

$$E_h \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} Q_h(e^{j\omega})|H(e^{j\omega})|^2 d\omega$$

$$E_f \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} Q_f(e^{j\omega})|F(e^{j\omega})|^2 d\omega \tag{5}$$

$$Q_h = \begin{cases} 0, & |\omega| < \pi/M \\ 1, & \pi/M < |\omega| < \pi \end{cases}, \quad Q_f = \begin{cases} 0, & |\omega| < 2\pi R - \pi/M \\ 1, & 2\pi/R - \pi/M < |\omega| < \pi \end{cases}$$

The distortion function we introduce is therefore given by the following weighted sum:

$$D = w_{al}E_{al} + E_{tr} + w_h E_h + w_f E_f \tag{6}$$

where the weight factors w_{al}, w_h, w_f are non-negative constants. Using Parseval's theorem D can be represented as a positive-semi-definite (PSD) quadratic form, in terms of the analysis and synthesis filter coefficients:

$$\begin{aligned} D &= 1 - 2\mathbf{f}^T\mathbf{q} + \mathbf{f}^T\mathbf{Q}\mathbf{f} + \text{terms which are independent of } \mathbf{f} \\ &= 1 - 2\mathbf{h}^T\tilde{\mathbf{q}} + \mathbf{h}^T\tilde{\mathbf{Q}}\mathbf{h} + \text{terms which are independent of } \mathbf{h} \end{aligned} \tag{7}$$

where $\mathbf{h}$ and $\mathbf{f}$ are vectors with elements obtained from consecutive terms of $h(n)$ and $f(n)$, respectively. The derivation of (7) and the expressions for the PSD matrices $\mathbf{Q}, \tilde{\mathbf{Q}}$ and the vectors $\mathbf{q}, \tilde{\mathbf{q}}$ are given in the Appendix.

3. ITERATIVE ALGORITHM FOR OPTIMAL FILTER-BANK DESIGN

When the analysis filter $h(n)$ is given, the synthesis filter $f(n)$ is computed by minimizing D, and vice-versa. Minimization of the corresponding PSD quadratic forms in (7) with respect to $f(n)$ or $h(n)$, gives the following linear equations for solving for the optimal filters, respectively:

$$\mathbf{Q}\mathbf{f}_{opt} = \mathbf{q}, \quad \tilde{\mathbf{Q}}\mathbf{h}_{opt} = \tilde{\mathbf{q}} \tag{8}$$

Assuming $w_h > 0$ and $w_f > 0$ (since we wish to limit the frequency magnitude response in the stop-band, both for $h(n)$ and $f(n)$), and following the derivation in the Appendix, we conclude that Q and $\tilde{Q}$ are PD matrices, yielding therefore unique solutions to (8).

If the given FIR filter is symmetric (i.e. linear phase), the corresponding computed filter from (8) is also symmetric. In this case, as shown in the Appendix, the dimension of the equation set can be reduced from $L \times L$ to $(L/2)\times(L/2)$, where L is the length of the computed filter.

The combined design of a pair of optimal analysis and synthesis filters is performed by an iterative algorithm, similar to [4], which converges to a local minimum of D, as follows:

a. Initialization: Let $\mathbf{f}^{(0)}$ be an initial synthesis filter, $\varepsilon > 0$ a threshold constant, $k=1$, $D^{(0)}=\infty$.

b. Given $\mathbf{f}^{(k-1)}$, compute $\mathbf{h}^{(k)}$ from (8): $\tilde{\mathbf{Q}}\mathbf{h}^{(k)}=\tilde{\mathbf{q}}$.

c. Given $\mathbf{h}^{(k)}$, compute $\mathbf{f}^{(k)}$ from (8): $\mathbf{Q}\mathbf{f}^{(k)}=\mathbf{q}$.

d. Compute the distortion D using (7).
If $(D^{(k-1)}-D^{(k)})/D^{(k)} < \varepsilon$, then go to step e.
Otherwise, $k \leftarrow k+1$, and return to step b.

e. Normalize $\mathbf{h}^{(k)}$ and $\mathbf{f}^{(k)}$ (as explained in the sequel). Stop.

By increasing the weights w_{al}, w_h, w_f in the distortion function defined in (6), the amplitude of the windows obtained by minimizing D, decreases. In order to restore the amplitude of the synthesised signal, without affecting the output SNR, $f(n)$ is scaled by a factor c_o, which minimizes the MSE relative to unity transmission:

$$E_{tr}(c) \triangleq \frac{1}{2\pi}\int_{-pi}^{\pi}\int_{-\pi}^{\pi} |e^{-j\omega n_o} - \frac{c}{MR}\sum_{l=0}^{M-1} F(e^{j\omega}W_M^{-l})H(e^{j\omega}W_M^{-l})|^2 d\omega \tag{9}$$

$$c_o = arg\{\min_c E_{tr}(c)\} \;;\; \mathbf{f}^{(k)} \leftarrow c_o \mathbf{f}^{(k)}$$

An expression for the factor c_o, in terms of the filter coefficients, is given in the Appendix.

The iterative algorithm consists of computing the analysis and synthesis filters alternately, minimizing the same distortion function. Hence, D is monotonically decreasing from iteration to iteration. The algorithm converges to a local minimum of D, which depends on the initial filter chosen.

4. DESIGN EXAMPLE

In this section, we present a filter design example, for a sub-band coder with M=16 complex-bands.

To achieve maximum coding efficiency, it is desirable to decimate the band-signals by the critical ratio $R=M$. However, the designed filters for the critical ratio, were not found to have the desired characteristics mentioned in section 1, unless they are of very long duration. Decreasing slightly the decimation ratio, using R=(M-1)=15, enables the design of filters which result in high performance SBC, but still of quite lower complexity than a conventional QMF coder of similar performance.

The impulse and frequency responses of the optimal filters, with M=16 bands, decimation ratio R=15, and length of 256 taps each, are shown in Fig. 3. The weight factors were chosen to be w_{al}=10, w_h=10, w_f=20, and the resulting filters provide good band-separation and a high performance SBC. The transfer function of the system in the above example (obtained from (1) by excluding the aliasing components) is given by:

$$T(\omega) \triangleq \frac{1}{MR}\sum_{l=0}^{M-1} H(e^{j\omega}W_M^{-l})F(e^{j\omega}W_M^{-l}) \tag{10}$$

and its magnitude is plotted in Fig. 4.

5. SBC IMPLEMENTATION

The SBC consists of the above analysis and synthesis filter banks, and adaptive quantizers with dynamic bit allocation.

The filter-banks are implemented efficiently, using the WOLA scheme [2]. The DFT is performed by the Decimation-In-Time FFT algorithm [5]. This algorithm, when used for transforming real sequences, has many redundancies. The FFT of a real sequence of length 16, requires only 12 real multiplies, 62 real adds, and the storage of only 3 constants (cosπ/4, cosπ/8, sinπ/8) [6]. Similarly, the IDFT is performed by the Decimation-In-Frequency FFT algorithm [6].

The M complex signals are uniquely represented by the following M real signals:

$$\begin{aligned} &Y_o(m) = X_o(x), \quad Y_{M-1}(m) = X_{M/2}(m), \\ &Y_{2k-1}(m) = \text{Re}\,X_k(m), \quad Y_{2k}(m) = \text{Im}\,X_k(m), k=1,2,\ldots,M/2-1 \end{aligned} \tag{11}$$

The real signals $Y_i(m), i=0,\ldots,M-1$, are quantized independently, using forward gain-adaptive quantizers [1]. Gain adaptation in the i-th band is based on the estimated and quantized variance $\hat{\sigma}_i^2$ of the signal $Y_i(m)$, which are updated every 16 samples of $Y_i(m)$ and are transmitted as side-information. The corresponding 16 samples of $Y_i(m)$, are quantized by a uniform quantizer [1], optimized for a zero-mean Gaussian PDF with variance $\hat{\sigma}_i^2$.

Based on the estimated variances $\hat{\sigma}_i^2$, the bit allocation is computed to minimize the MSE:

$$\begin{aligned} &E[\sum_{k=0}^{M-1} |X_k - \hat{X}_k|^2] = \\ &= E[Y_o - \hat{Y}_o]^2 + E[Y_{M-1} - \hat{Y}_{M-1}]^2 + 2E[\sum_{i=1}^{M-2}(Y_i - \hat{Y}_i)^2] \end{aligned} \tag{12}$$

where $\hat{X}$ and $\hat{Y}$ are the quantized versions of X and Y. The double weight of the errors in $\hat{Y}_i$, $i=1,\ldots,M-2$, is accounted for by multiplying the estimated variances $\hat{\sigma}_i^2, i=1,\dots,M-2$, by 2, before computing the allocation. A procedure for bit allocation is given in [7]. The allocation is also updated every N=16-samples of each of $Y_i(m), i=0,1,\dots,M-1$.

The computation rate required for implementing the analysis filter-bank is as follows: L multiplies, L−M adds and an FFT of length M for the analysis stage for producing M complex samples [2, Fig.7.19] where L is the filter length. The same number of operations is required for the synthesis stage [2]. For a decimation ratio of R=15, the implementation of both the analysis and synthesis filter-banks, requires 36 multiplies and 40 adds per input sample.

6. SIMULATION RESULTS

An SBC which utilizes the above optimized filters, and an SBC which is based on an 8-band QMF-bank, were simulated for a transmission rate of 16 Kbits/sec. The implementation of the QMF-bank requires 96 multiplies and 102 adds per input sample (using 32-tap filters). The DFT-based coder and the QMF-based coder were found to have similar subjective performance. Both coders yield high quality synthesized speech, degraded only by very slight hoarseness. The objective performance of the QMF-based coder is slightly higher: SNR=19.8 dB for the QMF SBC, as compared to 18.9 dB for the DFT-based coder. An attempt to reduce the QMF bank complexity to 48 multiplies per input sample, using shorter filters, caused noticeable degradation in the synthesized speech quality.

Another filter-bank structure is the generalized parallel QMF bank [8], [9], [10]. The filter-bank described in [8] has 8 bands, and requires about 75% of the number of multiplies and 150% of the additions as compared with the above uniform DFT filter-bank. However, in that filter-bank [8], adjacent bands are not well separated, and the coder using this filter-bank can be expected to have similar performance to the above DFT-based coder, only if longer filters will be used (thus increasing its complexity). The filter bank presented in [9] splits the signal into 16 bands (also not too well separated), and requires 42 multiplies and 82 adds per input sample. Both papers, [8] and [9], do not present comparisons to QMF-based or other coders. The SBC presented in [10], includes 5-band filter-bank, using longer filters (60 taps) to reduce the interband aliasing.

7. SUMMARY

A new approach for designing FIR filters for uniform DFT filter-banks, optimized for sub-band coding of speech was presented. A 16 Kbps SBC which utilizes the optimized filters was simulated and was found to achieve similar subjective performance as that of a QMF-based SBC, but effecting about 60% reduction in computations as compared to the QMF bank.

APPENDIX

1. Aliasing Distortion:
From (2), using Parseval's theorem:

$$E_{al} = \frac{1}{M^2R^2} \sum_{k=1}^{R-1}\sum_{n} \left| \sum_{l=0}^{M-1} \{ f(n)W_M^{ln} \} * \{ h(n)W_M^{ln}W_R^{-kn} \} \right|^2 \qquad (13)$$

Using the identity:

$$\sum_{k=0}^{M-1} W_M^{kn} = \sum_{k=0}^{M-1} e^{j2\pi kn/M} = M \cdot \delta(n \bmod M) = \begin{cases} M, & n=0 \\ 0, & n \neq 0 \end{cases} \qquad (14)$$

the following expression for E_{al} is obtained:

$$E_{al} = \frac{1}{R^2} \sum_s\sum_t\sum_n f(s)\,h(n-s)\,h(n-t)\,f(t) \cdot \cdot [R\ \delta((s-t) \bmod R)-1]\ \ \delta(n \bmod M) \qquad (15)$$

For simplicity we choose the filters $h(n)$ and $f(n)$ to be of equal length L, where L is an integer multiple of the transform size M. (15) can then be rearranged in either of the following two PSD quadratic forms:

$$E_{al} = \mathbf{f}^T \mathbf{Q}_{al} \mathbf{f} = \mathbf{h}^T \tilde{\mathbf{Q}}_{al} \mathbf{h}$$

$$\mathbf{f} = [f(0), f(1), \dots f(L-1)]^T$$

$$\mathbf{h} = [h(1), h(2), \dots, h(L)]^T$$

$$\mathbf{Q}_{al}(s,t) = \frac{1}{R^2} \sum_n h(nM-s)h(nM-t)[R\,\delta((s-t) \bmod R)-1], \quad 0 \le s,t \le L-1 \qquad (16)$$

$$\tilde{\mathbf{Q}}_{al}(s,t) = \frac{1}{R^2} \sum_n f(nM-s)f(nM-t)[R\,\delta((s-t) \bmod R)-1], \quad 1 \le s,t \le L$$

where $\mathbf{Q}_{al}$ and $\tilde{\mathbf{Q}}_{al}$ are matrices of dimension $L \times L$. Since $E_{al} \ge 0$, both symmetric matrices are PSD.

2. MSE relative to unity transmission:
If $h(n)$ and $f(n)$ are linear phase filters (both of length L), the delay of the analysis-synthesised system is n_o=L samples. Using Parseval's theorem and identity (14), we get from (3):

$$E_{tr} = 1-2\mathbf{f}^T\mathbf{q} + \mathbf{f}^T\mathbf{Q}_{tr}\mathbf{f} = 1-2\mathbf{h}^T\tilde{\mathbf{q}} + \mathbf{h}^T\tilde{\mathbf{Q}}_{tr}\mathbf{h}$$

$$\mathbf{q} = \frac{1}{R}\,\delta(n_o \bmod M)[h(n_o), h(n_o-1), \dots, h(n_o-L+1)]^T$$

$$\tilde{\mathbf{q}} = \frac{1}{R}\,\delta(n_o \bmod M)[f(n_o-1), f(n_o-2), \dots, f(n_o-L)]^T \qquad (17)$$

$$\mathbf{Q}_{tr}(s,t) = \frac{1}{R^2}\sum_n h(nM-s)\,h(nM-t), \quad 0 \le s,t \le L-1$$

$$\tilde{\mathbf{Q}}_{tr}(s,t) = \frac{1}{R^2}\sum_n f(nM-s)\,f(nM-t), \quad 1 \le s,t \le L$$

The symmetric matrices $\mathbf{Q}_{tr}$ and $\tilde{\mathbf{Q}}_{tr}$ can be shown to be PSD.

3. Side-Lobe Energy:
From (5), using Parseval's theorem:

$$E_h = \mathbf{h}^T\mathbf{Q}_h\mathbf{h}\ ;\ E_f = \mathbf{f}^T\mathbf{Q}_f\mathbf{f}$$

$$\mathbf{Q}_h(s,t) = q_h(s-t);\ 1 \le s,t \le L;\ q_h(n) = \mathcal{F}^{-1}\{\,Q_h(e^{j\omega})\,\} \qquad (18)$$

$$\mathbf{Q}_f(s,t) = q_f(s-t);\ 0 \le s,t \le L-1;\ q_f(n) = \mathcal{F}^{-1}\{\,Q_f(e^{j\omega})\,\}$$

Since $h(n)$ and $f(n)$ are FIR filters, E_h, $E_f > 0$, so both symmetric matrices $\mathbf{Q}_h$ and $\mathbf{Q}_f$ are PD. Combining (6),(16) and (18) we get equation (7), where $\mathbf{Q}$ and $\tilde{\mathbf{Q}}$ are given by:

$$\mathbf{Q} = w_{al}\mathbf{Q}_{al} + \mathbf{Q}_{tr} + w_f\mathbf{Q}_f$$

$$\tilde{\mathbf{Q}} = w_{al}\tilde{\mathbf{Q}}_{al} + \tilde{\mathbf{Q}}_{tr} + w_h\mathbf{Q}_h \qquad (19)$$

4. Case of Symmetric Filters:
If the given filter is symmetric, i.e. $h(t)=h(L+1-t)$, $t=1,\dots,L$, or $f(t)=f(L-1-t)$, $t=0,\dots,L-1$, then the matrices $\mathbf{Q}$ and $\tilde{\mathbf{Q}}$ in (19) satisfy:

$$\mathbf{Q}(s,t) = \mathbf{Q}(L-1-s, L-1-t), \quad s,t=0,1,\dots,L-1$$

$$\tilde{\mathbf{Q}}(s,t) = \tilde{\mathbf{Q}}(L+1-s, L+1-t), \quad s,t=1,2,\dots,L \qquad (20)$$

Following (20), we conclude that the computed filter obtained via (8) is also symmetric. Equation (8) can therefore be reduced to compute half the filter coefficients:

$$\mathbf{P}\mathbf{f}' = \mathbf{p}, \quad \tilde{\mathbf{P}}\mathbf{h}' = \tilde{\mathbf{p}}$$

$$\mathbf{P}(s,t) = \mathbf{Q}(s,t) + \mathbf{Q}(s, L-1-t), \quad s,t=0,\dots,L/2-1$$

$$\tilde{\mathbf{P}}(s,t) = \tilde{\mathbf{Q}}(s,t) + \tilde{\mathbf{Q}}(s, L+1-t), \quad s,t=1,\dots,L/2 \qquad (21)$$

$$\mathbf{h}' = [h(1), h(2), \dots, h(L/2)]^T, \quad \mathbf{f}' = [f(0), f(1), \dots, f(L/2-1)]^T$$

$$\mathbf{p} = [q(0), q(1), \dots, q(L/2-1)]^T, \quad \tilde{\mathbf{p}} = [\tilde{q}(1), \tilde{q}(2), \dots, \tilde{q}(L/2)]^T$$

5. Expression for c_0 (eqn. (9)):
Using Parseval's theorem and (14), $E_{tr}(c)$ can be expressed as:

$$E_{tr}(c) = 1 - 2c\,\mathbf{f}^T\mathbf{q} + c^2\mathbf{f}^T\mathbf{Q}_{tr}\,\mathbf{f} \qquad (22)$$

Minimizing $E_{tr}(c)$ results in:

$$c_o = (\mathbf{f}^T\mathbf{Q}_{tr}\,\mathbf{f}) \,/\, (\mathbf{f}^T\mathbf{q}) \qquad (23)$$

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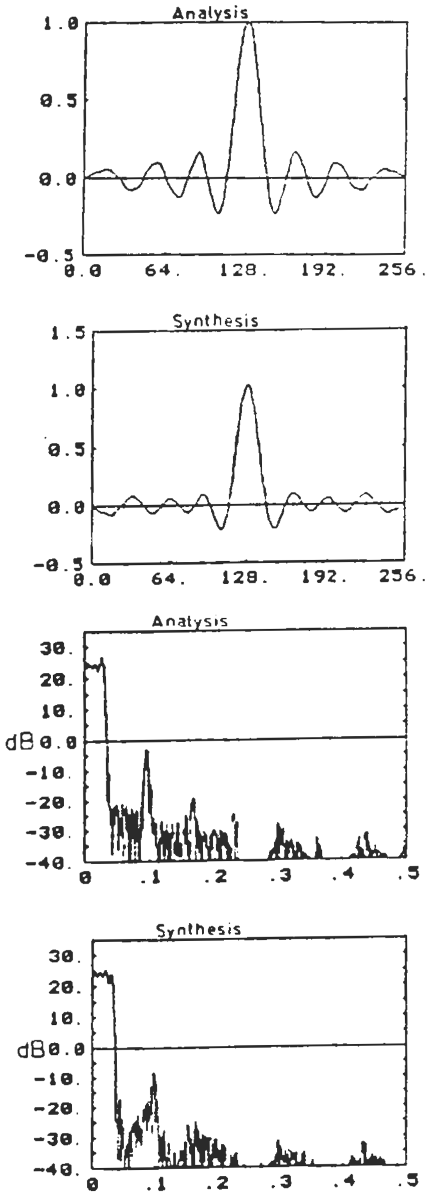

Fig. 3 The impulse and freqeuncy responses of the optimal filters

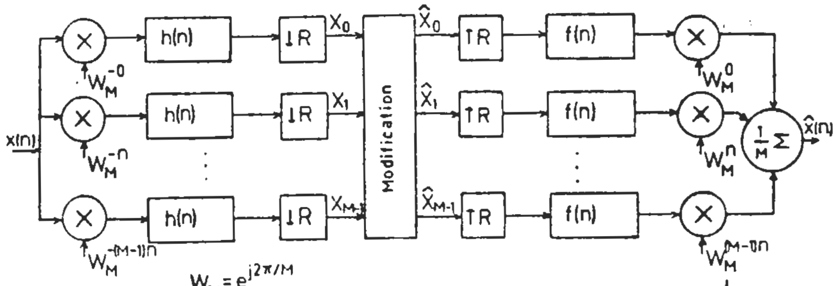

Fig. 1 The uniform DFT filter-bank

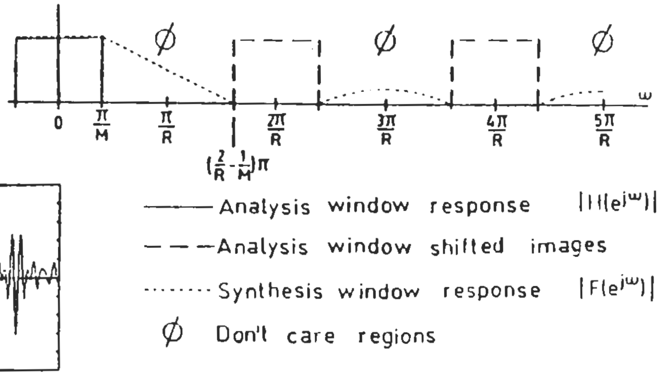

Fig. 2 The ideal filter frequency responses

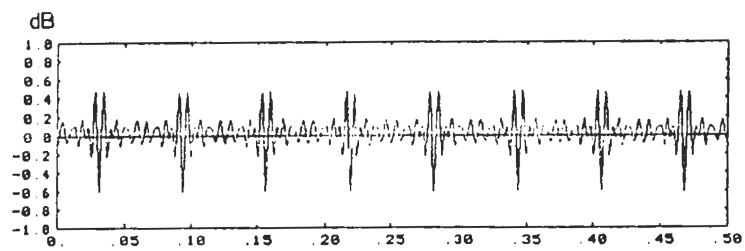

Fig. 4 The transfer function $|T(\omega)|$