MORPHOLOGICAL IMAGE CODING VIA BIT-PLANE DECOMPOSITION AND A NEW SKELETON REPRESENTATION

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ABSTRACT

A new approach for image coding based on bit-plane decomposition and binary morphological operations is presented. The image is first processed by an error-diffusion technique in order to reduce the number of bit-planes without a significant quality degradation. The bit-planes of the resulting image are converted to Gray code and are represented by a *modified morphological skeleton* which uses an *increasing* size structuring element. Redundancy in this representation is reduced with an algorithm motivated by a *Geometric Sampling Theorem* which we present. These reduced modified morphological skeletons are coded with an entropy coding scheme which was particularly devised for efficient skeleton coding. A post-processing operation, as well as the possibility of *geometric progressive transmission,* are also discused.

I. INTRODUCTION

Most of the research in image coding in the last three decades has concentrated on the use of orthogonal transformations or prediction models, which exploit the algebraic structure of the image. Recently, Maragos and Schafer [1] presented a scheme for binary image coding which exploits the geometry of these images via morphological skeleton representation. In this work we extend their approach for both binary and multilevel image coding.

The proposed image coding scheme consists of the following steps: First, the image is pre-processed by an error-diffusion technique in order to reduce the number of bit-planes, without significant quality degradation. The pixel values are subsequently represented in Gray-code, obtaining more uniform bit-planes. Next, each of the bit-planes is represented by means of a *modified* morphological skeleton which uses an *increasing* size structuring element instead of the single size one used in [1]. Redundancy in the modified skeletons is reduced based on a *Geometric Sampling Theorem*. These modified and reduced morphological skeletons, which are sparse representations of the bit-planes, are coded with a combination of different entropy coders particularly devised for efficient skeleton coding. These include Huffman coding of the number of consecutive lines having no skeleton points, and Elias coding followed by Ziv-Lempel universal coding for the remaining lines. Geometric errors in the bit-planes can also be introduced for bit-rate reduction. In such a case, post-processing operations for quality enhancement of the image, like random filling of undefined areas in the partially reconstructed bit-planes, are also suggested.

The remainder of this paper gives the underlying theory and describes in more details the coding algorithm. Section II introduces the modified morphological skeleton, and section III presents the Geometric Sampling Theorem. In section IV, description of all the coding steps and of the suggested geometric progressive transmission scheme is given. Experimental results are contained in section V, and a summary and conclusions in section VI.

II. MORPHOLOGICAL SKELETON

In the following, $A \ominus B$, $A \oplus B$, $A \circ B$, and $A \bullet B$ denote the basic morphological operations of erosion, dilation, opening, and closing, respectively, of a set A by a *structuring element* B (both A, B are in R^2 or Z^2) [2].

A. DEFINITIONS

Let X be a closed subset of R^2 such that the Convex Hull of X^C is equal to R^2 (a necessary condition for the exact reconstruction of X, see [2-6]) and such that the curvature $\kappa(\cdot)$ of the boundary of X, ∂X, is well defined everywhere, except at a finite number of points, where it may have only one sided tangents.

Let $D(x, \rho)$ be a closed disk of center x and radius $\rho \geq 0$ (in a two-dimensional Euclidian space). Then, a maximal disk in X and the skeleton of X are defined as follows:

Maximal Disk: A *maximal disk* $D(x, \rho_x)$ is one which is included in the object X, but not included in any other disk in X.

Skeleton: The *skeleton* $\psi(X)$ of an object X is defined as the family of centers of all maximal disks in X [5-7].

It is well known [3, 5, 6] that under the assumed conditions, X can be reconstructed from the set $\psi(X)$ together with the set of radii ρ_x (*skeleton pair*):

$$X = \bigcup_{x \in \psi(X)} D(x, \rho_x) \tag{1}$$

In a similar way we can define the skeleton $S^B(X)$ of a *discrete* subset X (a subset in Z^2) in relation with the *discrete structuring element* B, as the centers of all maximal structuring elements $(nB)_z$, where $(nB)_z$ represents the subset obtained after dilating B n-times and shifting the result by z; and $(nB)_z$ is maximal if and only if it is included in X and there is no other $(mB)_y$, $m > n$, such that $(nB)_z \subset (mB)_y \subset X$ [1, 2].

Lantuejoul [7] proved that the skeleton can be computed via basic morphological operations. We present the discrete version of the algorithm, as was used by Maragos and Schafer in [1] for binary image coding. Assume X to be a discrete subset, and B a discrete structuring element (i.e. subsets in Z^2), then the skeleton $SK^B(X)$ is given by [1, 2]:

$$SK^B(X) = \bigcup_{n=0}^{N(B)} S_n^B(X) \tag{2}$$

where

$$S_n^B(X) = (X \ominus nB) - (X \ominus nB) \circ B \ , \quad n = 0, 1, \ldots, N(B) \tag{3}$$

The subset $S_n^B(X)$ is called the n-th skeleton subset of X, computed with the structuring element B, and $N(B) = \max\{n : X \ominus nB \neq \emptyset\}$. The n-th skeleton subset $S_n^B(X)$ contains all the points $x \in X$ (and only those points), such that the element $(nB)_x$ is maximal in X.

Opened versions of X can be obtained via:

$$X \circ kB = \bigcup_{n=k}^{N(B)} S_n^B(X) \oplus nB \tag{4}$$

Hence, if $k = 0$ the original image is reconstructed.

B. MODIFIED MORPHOLOGICAL SKELETON

We propose a new morphological skeleton, for which the structuring element *size* increases with subsequent skeleton steps (n) (the shape is not changed). This new representation is motivated by the fact that when larger structuring elements are used, less skeleton subsets are obtained; enabling this way the increasing of the compression ratio (see section IV). In the modified morphological skeleton of X, each skeleton subset is computed with the largest possible structuring element; i.e. an element kB such that if $Y = X \circ kB$, then $Y \circ kB = Y$ and $Y \circ (k+1)B \subseteq Y$. From equation (4) we can see the follows:

$$X \circ B = X \circ 2B + S_1^B(X) \oplus B \quad (5)$$

Now, we can decompose $X \circ 2B$ in a different way:

$$X \circ 2B = \bigcup_{n=1}^{N(2B)} S_n^{2B}(X) \oplus n2B = X \circ 4B + S_1^{2B}(X) \oplus 2B \quad (6)$$

Where $S_n^{2B}(X)$, $n = 1, 2, \cdots, N(2B)$, are the skeleton subsets of X computed with the structuring element 2B (note that the union goes from $n=1$).

Subsequently, $X \circ 4B$ can be decomposed using $4B$ as a structuring element and this procedure can be continued using at each step a twice as big structuring element as in the previous step. We obtain this way the *modified morphological skeleton* $MS(X)$ [4]:

$$MS(X) = \bigcup_{n=0}^{N_M(B)} M_n(X) \quad (7)$$

where

$$M_0(X) = X - X \circ B = S_0^B(X) \quad (8a)$$

$$M_n(X) = (X \ominus 2^{n-1}B) - (X \ominus 2^{n-1}B) \circ 2^{n-1}B \quad , \quad n > 0 \quad (8b)$$

The skeleton subset $M_n(X)$ contains all the points $x \in X$ (and only those points) such that the element $(2^{n-1}B)_x$ is maximal in X; and $N_M(B) = \max\{n : X \ominus 2^{n-1}B \neq \emptyset\}$. We observe that with this modified morphological skeleton, less skeleton subsets are obtained due to the use of an increasing size structuring element, i.e., $N_M(B) = \lceil \log_2 N(B) \rceil < N(B)$.

The original image can be reconstructed from the modified morphological skeleton as follows:

$$X = \sum_{n=1}^{N_M(B)} [M_n(X) \oplus 2^{n-1}B] + M_0(X) \quad (9)$$

Opened versions of X can also be obtained if we omit some of the skeleton subsets. More details about this modified morphological skeleton are given in [4].

III. GEOMETRIC SAMPLING THEOREM

Let X be a set in R^2, satisfying the same conditions as in section II, and $Y_\rho = X \circ \rho B$ (where here B is selected to be the unit disk in R^2). Then, we define the following.

Singular Point: A skeleton point x, $x \in \psi(X)$, is a *singular point* if and only if there exists a point y in X such that the maximal disk $D(x, \rho_x)$ is the only one which contains it.

Let $S(X)$ be the set of singular points in X and let X^* be a subset of X defined by:

$$X^* = \bigcup_{x \in S(X)} D(x, \rho_x) \quad (10)$$

Boundary Singular Point: Similarly, we say that a point $x \in \psi(X)$ is a *boundary singular point,* if and only if there exists a point y on ∂X (the boundary of X) such that the maximal disk $D(x, \rho_x)$ is the only one which contains it.

We are interested in a set of points $\psi_m(X) \subseteq \psi(X)$, that we denote as the *minimal skeleton,* which guarantees exact reconstruction of X, and satisfies the condition that X can not be recovered from any subset of $\psi_m(X)$. Such a set exists, because in the worst case $\psi_m(X) = \psi(X)$. Similarly, we are interested in the minimal set for recovering ∂X.

For demonstration, Figure 1 shows a set in R^2 with its skeleton and singular points. In this case $\psi_m(X) = S(X)$.

All the proofs of the following results can be found in [4].

Lemma 1: All singular points are in $\psi_m(X)$, i.e., $S(X) \subseteq \psi_m(X)$.

Theorem 1: x is a *boundary singular point* if and only if x is a singular point.

This theorem means that a singular skeleton point contributes to X if and only if it contributes to ∂X.

Theorem 2 - The Geometric Sampling Theorem:

a) X^* covers all the X boundary, except for a finite number of points (a subset of the points with undefined curvature).

b) The subset of $S(X)$ with corresponding radii $r \geq \rho$, is enough for reconstruction of all the Y_ρ boundary, except for a finite number of points (these points are the same as in the first part of the theorem).

From the above two theorems we see the importance of the unique set $S(X)$ for the reconstruction of X, since each singular point contributes to ∂X and almost all of ∂X is covered by the maximal disks of $S(X)$. By the second part of Thm. 2 one is motivated to denote the morphological operation $X \circ \rho B$ as *a geometric low pass filter,* in analogy to the filter used in classical signal processing, where band-width is replaced here by the inverse of the radii of maximal disk. For an extension of the theorem, and an extended analysis of the analogy between this theorem and the classical Sampling Theorem, see [4].

In the case of discrete images (i.e. sets in Z^2), singular and boundary singular points can be defined in a similar way. Simulation results suggest that a discrete version of Thm. 1 may be valid. Concerning Thm. 2, a dual geometric sampling theorem for discrete images does not exist. However, as explained in the next section, the set of singular points plays an important role in the representation of discrete images as well.

IV. IMAGE CODING

Figure 2 presents the block diagram of the coding algorithm. We describe in this section each of the stages.

A. PRE-PROCESSING

The goal of the pre-processing stage is to represent the original image in a new form, more appropriate for our coding method.

The first step in this stage reduces the number of bit planes via the Floyd-Steinberg error diffusion algorithm [8]. We found that when reducing an 8-bit image to a 4-bit one with this method, reasonably good quality is obtained (in contrast with the poor quality obtained by simple truncation). With this technique we eliminate the least significant bit-planes of the 8-bit image, which due to their random-like structure, are typically difficult to compress. Thus, using 4 bit planes with error diffusion, the compression ratio is increased with no significant degradation. Subsequently, pixels in these bit-plane are represented in Gray-code, obtaining more uniform bit-planes which improves the coding algorithm performance.

The original image can also be pre-filtered with a two-dimensional 3×3 median filter or with a morphological filter, before the error diffusion step. On the basis of the Human Visual System properties [9], this filter improves the coder performance without significant changes in visual quality (see experimental results section).

B. BIT-PLANE REPRESENTATION

Each one of the four bit-planes obtained after the pre-processing stage is represented by means of the modified morphological skeleton described in the previous section (equations (7)-(9)). We use a 3×3 square as the basic structuring element (B). Usually, no more than 6 skeleton subsets were obtained for the different bit-planes ($N_M(B)$=5, see eqn. (9)).

C. SKELETON REDUCTION

Some skeleton points can be removed and exact reconstruction of the image from the reduced morphological skeleton can still be obtained. Maragos and Schafer [1] proposed an algorithm for redundant skeleton points elimination. However, their algorithm does not ensure that it always finds the *minimal* possible number of skeleton points which still allows reconstruction. Our approach for removing redundant skeleton points is based on their algorithm but is improved by using results from the Geometric Sampling Theorem described in the previous section. We found that the *singular points* of a discrete skeleton (defined in a similar way as the singular points of a continuous one), are not sufficient for exact image reconstruction, but they do reconstruct most of it (typically close to 90%). Therefore, the set of singular points is almost sufficient; and we have to care about the "optimal" coverage of only a small part of the image (typically 10%) instead of the optimal coverage of the whole image as needed in [1]. The resulting *search space* is much smaller than the original one, and a solution closer to the optimal one can be found with simpler methods. We decided to select the skeleton points, needed in addition to the singular points, according to the contribution of their corresponding maximal element to the partial reconstructed image (see equations (1), (4), and (9)). We denote by $M_n'(X)$ the reduced skeleton subset obtained from $M_n(X)$. We obtain this way that each of the bit-planes is being completely represented by its *reduced modified morphological skeleton (RMMS)*.

D. RMMS CODING

Each skeleton subset $M_n'(X)$, of the *RMMS* bit-plane representation, is coded as a binary image (these are very sparse binary images).

Two different entropy coding schemes are used, one for coding lines of the skeleton subsets having no skeleton points (empty lines), and another for the remaining lines. First, a Huffman code of the number of consecutive empty lines is generated. Then, the exact position of non-empty lines can be pointed out. The position of each skeleton point in its corresponding non-empty line, is coded by an Elias code [1] with four different symbols (three for the run-lengths of "zeros" expressed in ternary basis, and a *"comma"* symbol for the "ones", which represent skeleton points and separates the zeros runs). Therefore, two bits are needed for representation of these four symbols [1]. Finally, the binary output of the Elias code is compressed with the Ziv-Lempel universal coding algorithm.

This coding strategy was found to be very efficient for *RMMS* coding, because of their special structure as mentioned above. An improvement in the compression ratio was obtained using the *RMMS* representation instead of the original morphological skeleton representation proposed by Maragos and Schafer [1], mainly due to the reduction in the amount of bits needed for coding the empty lines.

By introducing geometrical errors in the different bit-planes, the compression ratio can be increased. These errors correspond to the omission of *RMMS* subsets $M_n'(X)$, $n \leq R \leq N_M(B)$, where R is selected according to the bit-plane importance (for more significant bit-planes, R is smaller or even zero); obtaining with this method smoothed versions of the bit-planes (equation (9), [4]). If just the smoothed bit-plane is coded, some points of X would appear in the reconstruction as part of the background. This causes considerable degradation of the subjective image quality. To circumvent this problem, we code the smoothed versions of both X and its complement, X^c, and subsequently fill-in randomly the undefined regions or "holes" (Fig. 3d).

Skeleton subsets that were initially omited can be progressively added in subsequent steps until the image quality is satisfactory, or the desired bit-rate is achieved. We denote this procedure as *Geometric Progressive Transmission* (see Fig. 4).

V. SIMULATION RESULTS

To evaluate the performance of the proposed coding scheme, it was simulated on a SUN 4/260, and a Gould IP8500 image display system was used. A woman's head and shoulder image (*"Lena"*) of size 512x512 pixels was used as a test image (for more examples see [4]). Performance of the algorithm is evaluated based only on the subjective quality of the images.

Fig. 3a shows the original image. Fig. 3b shows the image represented using only four bit-planes with error-diffusion. This image was coded at the rate of 0.35 bits per pixel (b/p), and represents what we call the "Four bits error-free image". The same picture, when represented via the morphological skeleton proposed in [1], instead of the *RMMS,* is coded at the rate of 0.40 b/p. Fig. 3c shows an image that was initially filtered with a 3x3 median filter followed by 4-bit error-diffusion. This image was coded at 0.29 b/p. Fig. 3d shows the reconstruction of an image in which the least significant bit-plane (of the 4-bit error-diffusion representation) was coded with no skeleton points of radii zero; both the image and the background were coded and "holes" were filled randomly (the remaining bit-planes were coded error-free). This image required 0.29 b/p.

Fig. 4 shows a simulation of the *Geometric Progressive Transmission* approach. In all the pictures, the two most significant bit-planes were coded error free. If X stands for the least significant bit-plane and Y for the next one, then: In Fig. 4a both X and X^c opened by $4B$, and Y and Y^c opened by $2B$ were coded. The bit-rate was 0.18 b/p. In Fig. 4b, the necessary points for error-free reconstruction of Y were added (i.e. skeleton points of radii zero and one), achieving a bit rate of 0.23 b/p. In Fig 4c, skeleton points with maximal disk $2B$ where added for the reconstruction of $X \circ 2B$ and $X^c \circ 2B$; and the bit rate is 0.25 b/p. The next step of the transmission can be seen in Fig. 3d, where $X \circ B$ and $X^c \circ B$ were coded (0.29 b/p).

Fig. 4d is similar to Fig. 3d, but some skeleton points of radius zero were added according to a subjective criterion [4]. The bit-rate is 0.30 b/p.

VI. SUMMARY AND CONCLUSIONS

In this paper an image coding scheme based on simple binary morphological operations is presented. The image bit-planes are represented by a modified morphological skeleton which uses an increasing size structuring element. Redundancy in this representation is reduced with an algorithm motivated by a Geometric Sampling Theorem. An entropy coding scheme was particularly devised for efficient coding of the reduced representation. The algorithm was developed both for error-free compression (of four bits images), and for compression with geometric errors in the bit-planes. The use of the proposed coding scheme for geometric progressive transmission is also considered.

The presented coding scheme can be seen as a first step in geometric coding of grayscale images. It is quite different from morphological approaches which are based on image segmentation and labeling (e.g. [9]) which cause undesired false contours. Also, in contrast to the quantization and blocking errors introduced by standard image coding algorithms, the error introduced by geometric deformations of the image is in general more pleasant to the observer.

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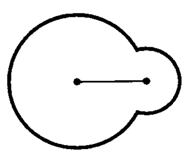

Fig. 1 - The two marked points are singular.

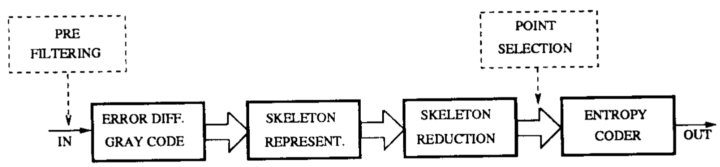

Fig. 2 - Block diagram of the coding scheme.

a b c d

Fig. 3

a b c d

Fig. 4