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# MORPHOLOGICAL MULTI-STRUCTURING-ELEMENT SKELETON AND ITS APPLICATIONS

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## ABSTRACT

*This work introduces a new binary image representation method - the Morphological Multi-Structuring-Element Skeleton (MSES), which is a generalization of the well-known Morphological Skeleton. Each point in the MSES representation is related to a combination of a finite number of pre-defined structuring-elements, in contrast to Regular-Skeleton points, which are related to a single scaled structuring-element. Its definition, properties and applications for image processing and coding are presented in this paper for the case of 2 structuring-elements.*

## I. INTRODUCTION

An increasing interest in the Morphological Skeleton representation has been observed recently, particularly for binary images [1]-[10]. The reason is the Skeleton's ability to extract relevant geometrical information about the shapes of objects in the image (considered by it as mathematical sets). This is done by decomposing the objects into the set-union of simpler shapes. Those shapes are maximal elements [2] taken from a pre-defined family, in which every element is a scaled version of one single basic element (called *structuring-element*). *Maximal elements* in an image object are defined in [1, 2] as the elements contained in the object in such a way that no bigger element contains it and is contained in the object.

The *size* of each maximal element, also called *the order of the skeleton point* related to it, has great significance. A *small* maximal element, for instance, may be seen as describing image detail or noise, whereas a *large* maximal element may be considered as a fundamental component of the image. Thus, image filtering can be done by discarding low-order skeleton points, and image analysis can be performed based on the high-order skeleton points.

On the other hand, the significance of the size is greatly dependent on the particular structuring-element chosen for the family generation; different structuring-elements may give different interpretations about the image parts. For instance, a square-like structuring-element may consider as noise what a diamond-like structuring-element might consider the corner of a fundamental component. Different filtering results and different shape analyses will be obtained for different structuring-elements.

The question of *"which structuring-element to choose?"* stimulated a great number of research works and different skeleton structures [2, 4, 6, 7, 8, 10].

The Morphological Multi-Structuring-Element Skeleton, proposed in this paper, does not calculate the optimum structuring-element, but it picks up, from a given finite set of shapes, the most appropriate *combination* of them *for each of the different parts of the image.* It does so by generating a multi-parameter family of shapes, instead of a 1-parameter family as in the Morphological Skeleton, and picking maximal elements from this family. The parameters of the family are the sizes of each of the pre-defined structuring-elements.

## II. The Multi-Structuring-Element Skeleton (MSES)

### *A. The Regular Morphological Skeleton*

Given a basic structuring-element $B$, a discrete family $\{A(n)\}_{n=0}^{\infty}$ of elements in $Z^2$, $n$ integer, is generated in the following way:

$$\begin{aligned} A(0) &= \{(0,0)\}, \\ A(n+1) &= A(n) \oplus B. \end{aligned} \tag{1}$$

where $\oplus$ means Morphological Dilation. (Definition of the morphological operations can be found in [1, 2]). It is easy to see that

$$A(n) = \underbrace{B \oplus B \oplus ... \oplus B}_{n\ times} = nB. \tag{2}$$

An example of such a family is shown in Fig. 1(a), where $B$ is a unit square. The family $\{A(n)\}$ is the set of all discrete squares.

The Morphological Skeleton, $SK(X)$, of a binary image $X$ (in relation to the family $\{A(n)\}$) is the set-union of its $n$-th order skeleton subsets which are defined, for the discrete case (images in $Z^2$), by:

$$S_n(X) = Y_n - Y_n \circ B, \tag{3}$$

where

$$Y_n = X \ominus A(n). \tag{4}$$

with $\ominus$ denoting Morphological Erosion; $\circ$ – Morphological Opening and the minus sign – set-subtraction [2].

Fig. 2(a) shows a shape ($X$) and its Skeleton ($SK(X)$), based on the discrete 1-parameter family shown in Fig. 1(a).

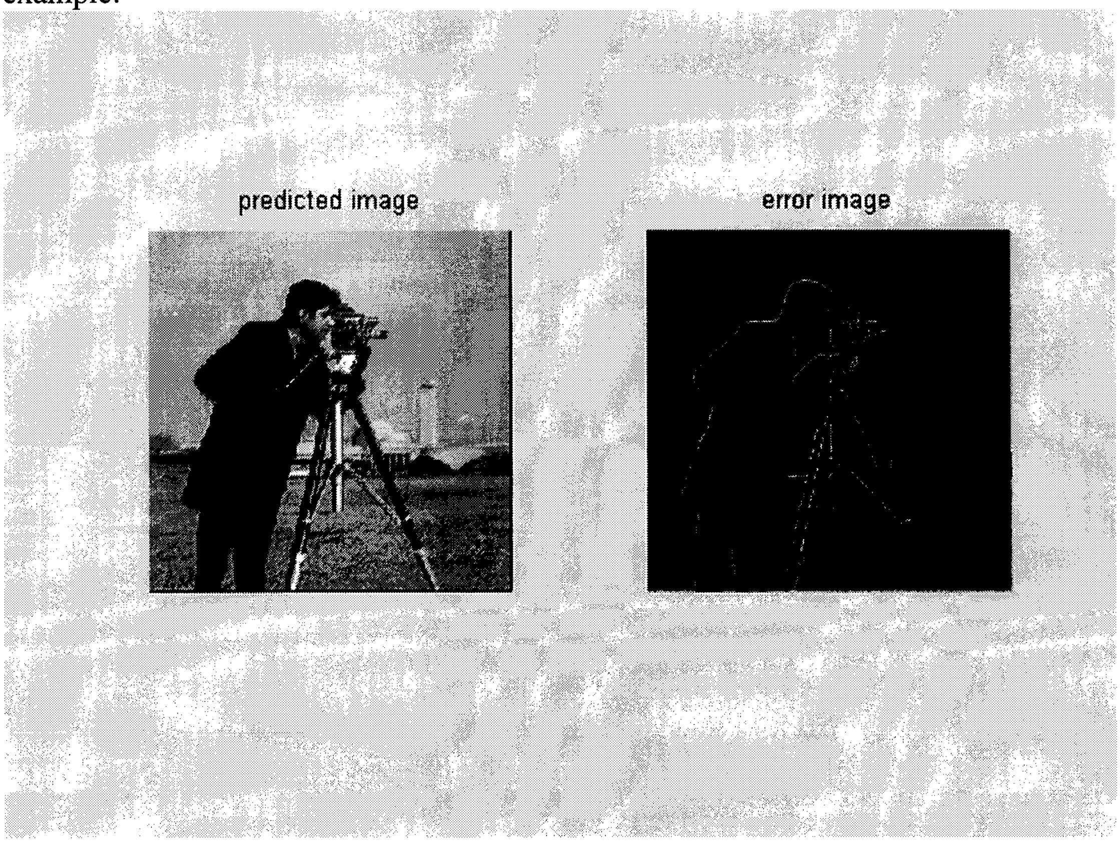
The set $S_n(X) \oplus A(n)$ is the union of *all the maximal elements of size n* of the image $X$. It is easy to show that

$$\bigcup_{n=k}^{\infty} S_n(X) \oplus A(n) = X \circ A(k), \tag{5}$$

13.

```
function [med_im,err_im] = med(im);

sz = size(im);

a_im = filter2([0 0;1 0],im);
b_im = filter2([0 1;0 0],im);
abc_im = filter2([-1 1;1 0],im);

comb_im = zeros(sz(1),sz(2),3);
comb_im(:,:,1) = a_im;
comb_im(:,:,2) = b_im;
comb_im(:,:,3) = abc_im;

med_im = median(comb_im,3);
err_im = im - med_im;
```

example:



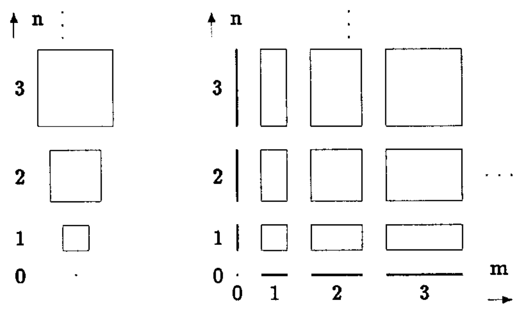



Figure 1: (a) A 1-parameter (size) family of shapes generated by the Regular Skeleton. (b) A 2-parameter family, generated by the MSES.

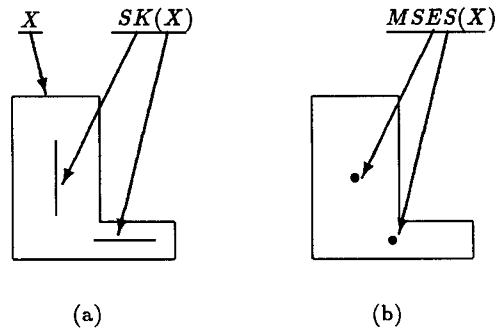



Figure 2: (a) A shape $X$ and its Skeleton $SK(X)$, related to the family shown in Fig.1(a). (b) Same shape $X$ and its MSES, related to the family shown in Fig.1(b).

which means that morphological filtering can be done by simply discarding small-order skeleton subsets. Error-free reconstruction of the image $X$ can be obtained from (5) by setting $k$=0.

*B. Definition of the MSES*

The MSES may be defined for any finite number of structuring-elements. For simplicity, we define it in this paper for the case of 2 structuring-elements only; the generalization is simple.

Given 2 structuring-elements, $B_1$ and $B_2$, they generate a family $\{A(n,m)\}_{n,m=0}^{\infty}$ of elements, $n$,$m$ integers, in the following way:

$$A(n,m) = nB_1 \oplus mB_2. \qquad (6)$$

The above family has the following property:

$$A(n,m) = A(n-1,m) \oplus B_1 = A(n,m-1) \oplus B_2. \qquad (7)$$

for $n, m > 0$.

Fig. 1(b) shows an example of such a family, for the case that $B_1$ is a vertical *unit line* and $B_2$ is an horizontal *unit line*. The family $\{A(n,m)\}$ is the set of all the discrete rectangles. Notice that the 1-parameter family shown in Fig. 1(a) is contained in the 2-parameter family just defined.

We define the Morphological Multi-Structuring-Element Skeleton, $MSES(X)$, for the above family, as the union $MSES(X) = \bigcup_{n=0}^{\infty} \bigcup_{m=0}^{\infty} S_{n,m}(X)$, where

$$S_{n,m}(X) = Y_{n,m} - [(Y_{n,m} \circ B_1) \cup (Y_{n,m} \circ B_2)], \qquad (8)$$

$$Y_{n,m} = X \ominus A(n,m). \qquad (9)$$

$MSES(X)$ is the set of center points of the maximal elements in $X$ from $\{A(n,m)\}$. In other words, the MSES automatically matches the different regions of the image with each of the structuring-elements and decides which one, *or which combination of them*, best represents it. Each of its points is related to 2 parameters; the first one ($n$) is associated with the size of the first structuring-element and the second one ($m$) with the size of the second structuring-element.

Fig. 2(b) shows $MSES(X)$ (same shape $X$ as in Fig. 2(a)), based on the discrete 2-parameter family shown in Fig. 1(b). It contains only *two points*, because $X$ is composed of only 2 rectangles. The point related to the large rectangle belongs to $S_{6,3}(X)$ and the other one to $S_{1,6}(X)$. Every other subset of $MSES(X)$ is empty.

An equation similar to (5) is obtained for the MSES:

$$\bigcup_{n=j}^{\infty} \bigcup_{m=k}^{\infty} S_{n,m}(X) \oplus A(n,m) = X \circ A(j,k). \qquad (10)$$

The original image can be fully reconstructed from the collection $\{S_{n,m}(X)\}$ if we set $(j,k) = (0,0)$ in equation (10).

Also from (10), one obtains that:

$$\bigcup_{(n,m)\neq(0,0)} S_{n,m}(X) \oplus A(n,m) = (X \circ B_1) \cup (X \circ B_2), \quad (11)$$

which is the result of discarding the lowest-order skeleton subset ($S_{0,0}(X)$).

The Regular Morphological Skeleton is a particular case of the MSES. It is obtained by choosing $B_1 = B$, and any $B_2$ so that $X \ominus B_2 = \emptyset$. Such choices yield $S_{n,m}(X) = \emptyset$ for all $m \geq 1$, and $S_{n,0}(X) = S_n(X)$.

*C. The Minimal MSES*

The Regular Skeleton is normally a redundant representation [2, 5, 10]. I.e., one can remove some skeleton points and still recover the original image with no error. Motivated by this observation, some authors have proposed algorithms [2, 5] for finding a *Minimal Skeleton* which is defined in [2] as a subset of the skeleton points which is sufficient for reconstructing the whole image, but not so if any of its points is removed. A Minimal Skeleton always exists and may be not unique.

Like the Regular Skeleton, also the MSES is a redundant representation, and a *Minimal MSES* may be defined in the same way as the Minimal Skeleton above. The same algorithms for finding Minimal Regular Skeletons can be used for finding a Minimal MSES, which fully represents the original image and contains no redundant points. Its existence is also assured, and it is usually not unique, too.

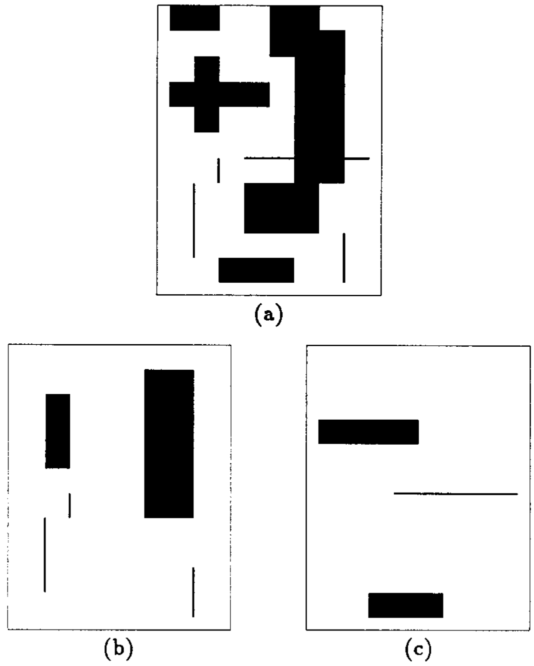


Figure 3: Shape Classification via Minimal MSES: (a) An image composed of rectangular features. (b) Features for which $R \geq 3$. (c) Features for which $R \leq 1/3$.

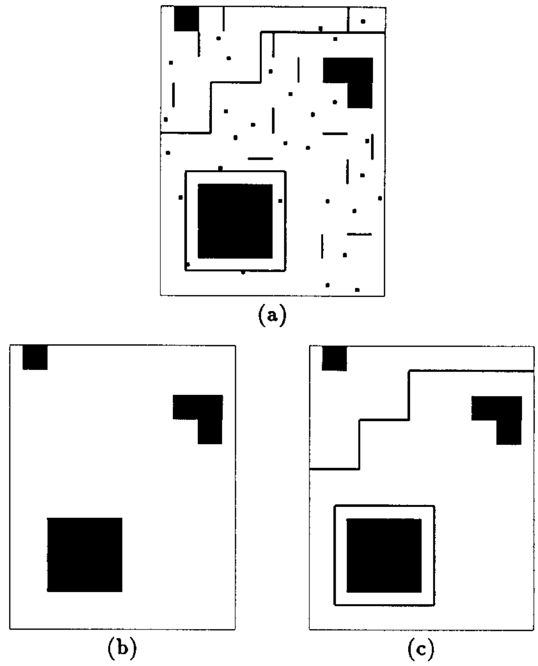


Figure 4: Image Filtering: (a) Noisy image. (b) Filtered reconstruction from the Regular Skeleton ($n \geq 1$). (c) Filtered reconstruction from the MSES ($n + m \geq 2$).

## III. Characteristics and Applications of the MSES

### *A. Size Ratio and Shape Classification*

Let us consider a *Minimal MSES* representation of a binary image composed by several objects. By considering the relation between the parameters $n$ and $m$ associated to the Minimal MSES points, one can decide whether the image objects, or their parts, are similar to $B_1$, $B_2$, or none of them.

Let the *size ratio* of an MSES point be defined as $R = n/m$. If $R$ is greater or equal to a given $R_0 > 1$, then the maximal element corresponding to the MSES point is similar to $B_1$. If $R$ is smaller or equal to $1/R_0$, then the maximal element is similar to $B_2$. Otherwise, the element doesn't look like any of the 2 structuring-elements.

Fig. 3(a) shows an image $X$ composed of several rectangular features. A Minimal MSES representation of $X$, with the rectangular family defined in the last section, was calculated. Fig. 3(b) shows maximal elements corresponding to Minimal MSES points for which $R \geq 3$. Those features are very similar to $B_1$ (vertical line). Fig. 3(c) shows maximal elements corresponding to Minimal MSES points for which $R \leq 1/3$. Those features are very similar to $B_2$ (horizontal line).

### *B. Partial Reconstruction and Image Filtering*

Discarding lower-order Regular Skeleton subsets in the reconstruction process produces smoothed versions of the original image, as shown in equation (5). Equations (10) and (11), for the MSES, provide a greater variety of options for filtering the original image.

One can see that discarding $S_{0,0}(X)$ alone (equation (11)) produces the same result as the union of the openings by the 2 structuring-elements, which is a more selective filter than the opening by a single structuring-element. Generally, for an MSES with $N$ structuring-elements (instead of 2), the removal of the lower MSES subset gives the union of the openings by each of the $N$ structuring-elements, which was shown to have good noise cleaning properties in [9].

Many other morphological filters can be obtained by removing different combinations of lower-order subsets. Fig. 4 shows the result of removing the MSES subsets with $n+m < 2$, in contrast to the result of removing the Regular Skeleton subsets with $n < 1$.

All the morphological filters that may be obtained removing Regular Skeleton subsets may also be obtained through the MSES, by removing the appropriate subsets.

### *C. Compact representation and Error-Free Image Coding*

If $B = pB_1 \oplus qB_2$, for any integers $p,q \geq 0$, then the 1-parameter family generated by $B$ is contained in the 2-parameter family generated by $B_1$ and $B_2$. In this case, the number of representation points in a Minimal MSES (using $B_1$ and $B_2$) is never greater than the number of points in a Minimal Regular Skeleton (using $B$). If we take as an example the families from Fig. 1, and an image composed only by squares, and not by rectangles, then its two skeleton representations are identical, with the same number of points. Normally, though, an image is composed of elements from the 2-parameter family that are not contained in the 1-parameter family, and hence its Minimal MSES has fewer points than its Minimal Regular Skeleton.

Simulation tests for *error-free* coding of Minimal MSES and Minimal Regular Skeleton representations were performed. The binary images used for the tests are the Most Significant Bit-Plane of some grayscale images. The results are presented in Table 1, both in terms of number of representation points, and bits/pixel of the compressed image. The compression method used is Huffman Coding of the Runlengths of the "0" sequences. It shows that the Minimal MSES has fewer points than the Minimal Regular Skeleton (SK), and also that it achieves better compression rates.

| Picture (MSB) | # points | | | Bits/Pixel | |
|---|---|---|---|---|---|
| | SK | MSES | Savings | SK | MSES |
| Lena | 12607 | 8017 | 36% | 0.32 | 0.30 |
| House | 1079 | 639 | 41% | 0.12 | 0.11 |

Table 1

*D. Pattern Spectrum and Image Analysis*

The Regular Skeleton is closely related to a *morphological pattern spectrum* [10]. The pattern spectrum conveys geometrical information which can be further analyzed and processed. The MSES, because of its multiparameter structure, may be seen as closely related to a *multi-dimensional pattern spectrum*, which contains the 1-dimensional pattern spectrum and conveys more and finer details for analysis.

The discrete morphological pattern spectrum is defined in [10] as:

$$PS_n^B(X) = \#[X \circ nB - X \circ (n+1)B].$$

where $\#(\cdot)$ denotes finite set cardinality. The same type of generalization used to generate the *2-element MSES* may be used to define a *2-dimensional pattern spectrum*:

$$PS_{n,m}^{B_1,B_2}(X) = \#\{X \circ (nB_1 \oplus mB_2) - (X \circ [(n+1)B_1 \oplus mB_2]) \cup (X \circ [nB_1 \oplus (m+1)B_2])\}$$

## IV. Generalizations of the Minimal MSES

If the Regular Skeleton has $N$ subsets, than the MSES with 2 structuring-elements has approximately $N^2$ subsets. This causes the MSES to demand much more computation time than the Regular Skeleton. On the other hand, a *Modified Skeleton* with a *Geometrical-Step* family of elements $A(n) = (2^n - 1)B$ [4, 7, 8] gives a skeleton with only about $\log_2 N$ subsets. It was shown in [4] that not only is the Modified Skeleton faster to compute than the Regular Skeleton, it is also more compact *in terms of bit-rate*. A *Modified MSES*, with $A(n,m) = (2^n - 1)B_1 \oplus (2^m - 1)B_2$, may also be defined as in the case of the Regular Skeleton, and will produce $(\log_2 N)^2$ skeleton subsets, which is usually smaller than $N$.

Table 2 shows the results of an error-free coding simulation using a Minimal Modified Regular Skeleton and a Minimal Modified MSES.

| Picture (MSB) | # points | | | Bits/Pixel | |
|---|---|---|---|---|---|
| | Modif. SK | Modif. MSES | Sav. | Modif. SK | Modif. MSES |
| Lena | 13231 | 8929 | 33% | 0.31 | 0.27 |
| House | 1261 | 703 | 44% | 0.12 | 0.10 |

Table 2

The *Modified Skeleton* and the Regular Skeleton itself are particular cases of the *Generalized-step Morphological Skeleton* defined in [7]. The Generalized-step Morphological Skeleton family of elements is generated not from a single structuring-element but from a series of elements $\{B(n)\}$, such that $A(n) = A(n-1) \oplus B(n)$. A *Generalized-step MSES* may be defined in the same simple way – its element-family is generated from the families $\{B_1(n)\}$ and $\{B_2(m)\}$ by $A(n,m) = A(n-1,m) \oplus B_1(n) = A(n,m-1) \oplus B_2(m)$. The Generalized-step MSES *is a further generalization* of the Generalized-step Morphological Skeleton, since the latter one is a particular case of it.

The MSES can be generalized also to grayscale images in the same way it has been done for the Regular Skeleton [3].

## V. Conclusions

A new image representation was presented which generalizes the Morphological Skeleton in such a way that it represents image details in a more descriptive and efficient way. In applications such as image coding, filtering and analysis, it was demonstrated that the new skeleton has an advantage over the regular one.

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