Math. Morph. Workshop, Barcelona
May 93

pp. 133-138

TWO-SIDED SKELETON
- A Representation Composed of Both Positive and Negative Morphological Elements

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ABSTRACT

This work presents a novel morphological representation structure - the Two-Sided Morphological Skeleton. *It represents a shape not only by the centers of* "positive elements" *(foreground features), as the ordinary Morphological Skeleton does, but also by the centers of* "negative elements" *(background features, such as* holes*). I.e., it represents an image by elements from both sides of the Pattern Spectrum.*

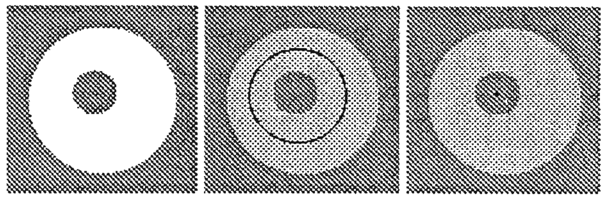
The Two-Sided Morphological Skeleton can be a very efficient tool in areas such as Multi-Resolution Representation, Shape Analysis and Pattern Recognition, since negative elements are as much important to image comprehension as positive elements. It has also a potential in Coding, because it is a compact error-free representation of the original image.

In this work, the Two-Sided Morphological Skeleton is defined and studied for both binary and grayscale images.

I. INTRODUCTION

The Morphological Skeleton has been successfully applied in many Image Processing application areas for efficient shape representation, and as a feature extractor and classifier (according to size). However, it does not take directly into account background features such as "holes" and "negative shapes", making it less efficient in such cases. A simple example is shown in Fig. 1: Fig. 1(a) is a binary picture, where the bigger white circle is a "positive shape" and the smaller black circle is a "negative shape"; its ordinary Morphological Skeleton, calculated with a disk as structuring-element, is a circle between the two circular edges as shown in Fig. 1(b). There is a need for more efficient representations that consider both positive and negative disks, which would represent the same image with just two points (the center of each circle), as shown in Fig. 1(c), by the black and white dots.

The basic morphological operators which are able to extract both positive and negative features from given shapes are the Opening-Closing and the Closing-Opening filters. Several representation structures have been proposed based on these operators, e.g., [1-3].



(a) (b) (c)

Fig. 1: (a) A binary image, (b) its Morphological Skeleton, (c) a more natural representation of the image.

Toet's "band-pass" pyramid [1], for instance, is an error-free representation where the Closing-Opening filter replaces the LPF in linear pyramids, prior to the decimation process, and either a Closing or a Dilation filter replaces the LPF needed in the interpolation process. This pyramid does produce a two-sided error-free representation, in which both positive and negative features are selected and classified according to resolution. On the other hand, Toet's pyramid levels contain not only genuine image features, extracted according to size by the Closing-Opening filter, but also spurious features, originated from by down-sampling process [2]. This is because subsampling a morphologically filtered image is not an invertible process [4, 5]. The genuine features and the spurious ones are indistinguishable, as demonstrated in Fig. 2. Fig. 2(a) shows a grayscale image and Fig. 2(b) shows its genuine morphological features, obtained at each step by the difference of the images in the input and output of the Closing-Opening decimation filter. Fig.2(c) shows the related Toet's "band-pass" pyramid, which contains the features from Fig. 2(b) plus spurious features. The pyramid in Fig. 2(b) is not error-free, and the non-morphological spurious features of Toet's pyramid compensate its lossy nature. Throughout this paper, all the "band-pass" pyramidal levels are represented with a shift of 128 in their graylevels, so that negative features can also be shown. The structuring-function used is a 2×2 flat square.

To avoid the generation of spurious features, Zhou and Venetsanopoulos suggested in [2] a different pyramidal representation based on Alternating Sequential Filters (ASF), with no down-sampling. ASF's were in-

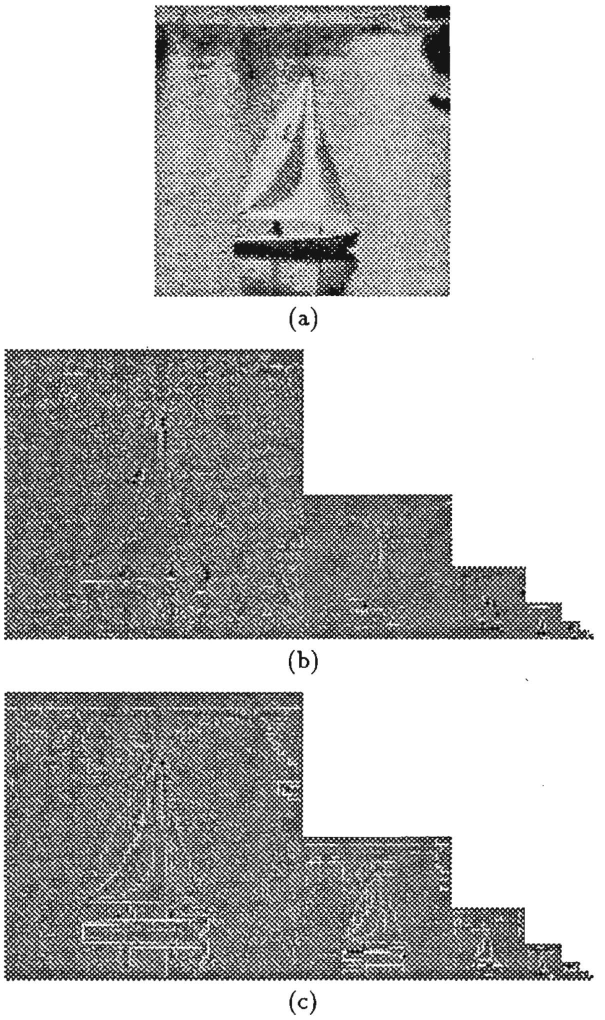


Fig. 2: (a) A 128×128 grayscale image, (b) genuine morphological features, extracted by the Opening-Closing filter at each step, (c) Toet's pyramid, with spurious features originated by the down-sampling process.

troduced by Sternberg [6], and analyzed by Serra [7, chapter 10]. An ASF is obtained by the composition of Opening-Closing (or Closing-Opening) filters, each one using an element bigger than the one in the previous stage. ASF's have been used extensively in image filtering.

In [2, 3], the ASF-based "low-pass" pyramid $\{f_n\}$ was generated in the following way:

$$\begin{array}{l} f_n \equiv (f_{n-1} \circ n \cdot g) \bullet n \cdot g, \quad n \geq 1 \\ f_0 \equiv f \end{array} \qquad (1)$$

where $f = f(x,y)$ is the original image, $g = g(x,y)$ is a pre-defined structuring-function, $\oplus$, $\ominus$, $\circ$ and $\bullet$ denote, respectively, grayscale dilation, grayscale erosion, grayscale opening and grayscale closing, and $n \cdot g$ stands for $g(x,y)$ dilated $n-1$ times by itself.

The related "band-pass" pyramid, called *Feature-Width Morphological Pyramid* [2], was defined as the difference between each level f_n and the next level f_{n+1}. Fig. 3 shows the first four levels of the Feature-Width Pyramid.

It is indeed an error-free representation of the image, which takes into account both positive and negative elements, and does not have the disadvantage caused by down-sampling, as in Toet's pyramid. On the other hand, the extracted features (both positive and negative) in level n of this "band-pass" pyramid have width equals to n, as seen in Fig. 3. Which means that for an efficient representation, as needed in coding, and for precision in determining the position of the features, as needed in pattern recognition, *thinning must be performed.* The thinning, which is obtained through erosion, *is not invertible*, thus producing spurious features like the down-sampling in Toet's pyramid.

The Two-Sided Morphological Skeleton presented in this paper is an invertible process, providing a thinned alternative to Zhou and Venetsanopoulos's pyramid. It is an error-free representation, with no spurious features, formed by the centers of both the positive and the negative elements extracted at each level of the ASF-based pyramid defined in (1).

II. TWO-SIDED SKELETON

In this paper we consider only discrete pictures, i.e., sets in Z^2 (binary discrete pictures) or functions over Z^2 (grayscale discrete pictures). The definition and properties of the Two-Sided Skeleton can be extended to continuous pictures (sets or functions over R^2), but this extension is not in the scope of this paper.

A. Binary Pictures

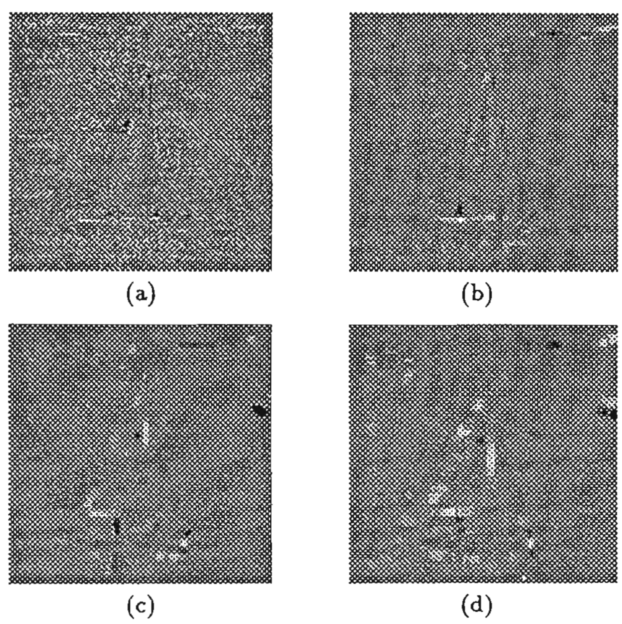


Fig. 3: (a)-(d) The first four levels of the Feature-Width Morphological Pyramid of the image shown in Fig. 2(a).

In the binary case, the ASF-based "low-pass" pyramid is defined as follows:

$$X_n = (X_{n-1} \circ nB) \bullet nB, \quad n \geq 1 \qquad (2)$$
$$X_0 = X$$

where, X is the original shape, B is the structuring-element, $\circ$ and $\bullet$ are respectively the binary opening and the binary closing operations.

As opposed to the thinned version of the Feature-Width Pyramid, in which the levels are obtained by *first* taking the difference between consecutive "low-pass" pyramid levels, and *then* performing *thinning*; the Two-Sided Skeleton performs *first* the thinning of each "low-pass" pyramid level and *then* takes the difference. The formal definition is as follows.

The Two-Sided Skeleton of a set $X \subseteq Z^2$ with a given structuring-element $B \subseteq Z^2$ is two collections of sets $\{S_n^+\}_{n=0}^{\infty}$ and $\{S_n^-\}_{n=0}^{\infty}$, where S_n^+ is called *the Positive Skeleton Subset of order n* and S_n^- is called *the Negative Skeleton Subset of order n*. For every natural n, S_n^+ and S_n^- are given by:

$$S_n^- \triangleq (X_{n+1} \oplus nB) - (X_n \oplus nB) \qquad (3)$$

$$S_n^+ \triangleq (X_n \ominus nB) - (X_{n+1} \ominus nB) \qquad (4)$$

where $\{X_n\}$ is the "low-pass" pyramid defined in (2). In (3) and (4), $\oplus$ denotes binary dilation, $\ominus$ denotes binary erosion, and the minus-sign denotes here the *set-difference* operation. Note that the thinning of *negative* features is obtained by *dilation*, whereas the thinning of *positive* features is obtained by *erosion*.

The *Positive* Skeleton Subsets $\{S_n^+\}$ correspond to the positive side of the Pattern Spectrum; they contain the centers of positive features that represent the original image, where by *feature* we mean a dilated and translated version of the structuring-element. The *Negative* Skeleton Subsets $\{S_n^-\}$ correspond to the negative side of the Pattern Spectrum; they contain the centers of negative features.

Let us define *the Positive Skeleton* of a shape as the union of all its *positive* skeleton subsets, and *the Negative Skeleton* as the the union of all its *negative* skeleton subsets. Figures 4(c) and 4(d) show the Positive and the Negative Skeletons, respectively, of the shape in Fig. 4(a). If we compare them with the ordinary Morphological Skeleton, shown in Fig.4(b), we notice that they represent the shape in a more meaningful and efficient way.

The Two-Sided Skeleton subsets fully represent the original image X. As shown in the Appendix, every level n of the pyramid defined in (1) can be recovered from the lower-resolution level $n+1$ by "adding" the information in the positive and the negative skeleton subsets of order n in the following way:

$$X_n = \{[X_{n+1} - (S_n^- \oplus nB^s)] \circ nB \cup (S_n^+ \oplus nB)\} \bullet nB \qquad (5)$$

where B^s denotes the set symmetric to B: $B^s \triangleq \{-b \mid b \in B\}$. Fig. 5 shows the block diagrama of the reconstruction process (5).

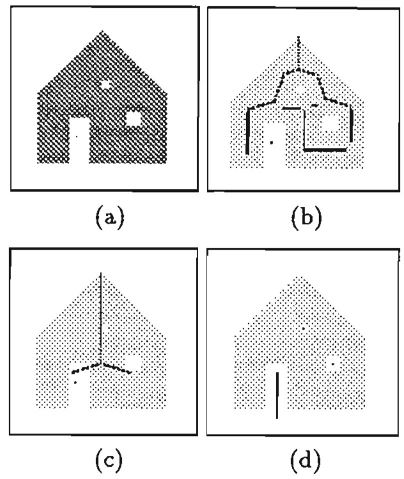


Fig. 4: The Two-Sided Skeleton versus the ordinary Skeleton. (a) A binary Image, (b) its ordinary Morphological Skeleton with a 3x3 square as structuring-element, (c) its *positive* and (d) its *negative* Two-Sided Skeleton subsets with the same structuring-element.

Since, for a bounded X, there exists a natural N such that $\forall n \geq N$, $X_n = \emptyset$, all the information is retained in the sets $\{S_n^+\}_{n=0}^{N-1}$ and $\{S_n^-\}_{n=0}^{N-1}$. By applying (5) successively from $n = N-1$ down to 0, the original image $X = X_0$ is reconstructed.

A partial reconstruction can be obtained by applying (5) from $n = N-1$ down to a given number $k > 0$. The image obtained by this process is the pyramid level X_k, which is a smoothed version of the original image.

B. Grayscale Pictures

A Two-Sided Skeleton may also be defined for a function $f(x,y)$, with a given structuring-function $g(x,y)$. The Positive and Negative Skeleton Function of order n, $s_n^+(x,y)$ and $s_n^-(x,y)$, respectively, are defined as follows:

$$s_n^- \triangleq \begin{cases} (f_{n+1} \oplus n \cdot g) - (f_n \oplus n \cdot g), & \text{if positive} \\ 0, & \text{otherwise} \end{cases} \qquad (6)$$

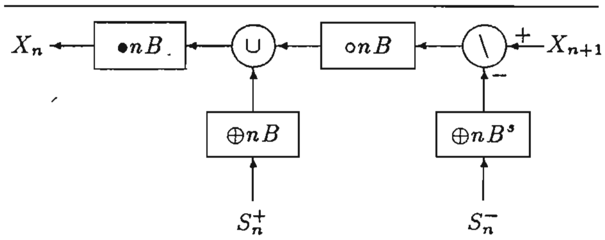


Fig. 5: Block diagram of the recursive reconstruction process. ("\" denotes *set-difference*).

$$s_n^+ \triangleq \begin{cases} (f_n \ominus n \cdot g) - (f_{n+1} \ominus n \cdot g), & \text{if positive} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

where $\{f_n\}$ is the "low-pass" ASF-based pyramid defined in (1). In (6) and (7), $\oplus$, $\ominus$ denote, respectively, grayscale dilation, and grayscale erosion. Note that all the Positive and Negative Skeleton Functions have non-negative values.

Fig. 6(a) shows a grayscale picture $f(x, y)$ of size 128×128 pixels (the same one as shown in Fig. 2(a)), Fig. 6(b)-(e) its skeleton functions for $n = 0, \ldots, 3$, and Fig. 6(f) the 4^{th} level of the pyramid defined in (1). The image functions in Fig. 6(b)-(e) were obtained by the formula $s_n^+(x, y) - s_n^-(x, y) + 128$, for $n = 0, \ldots, 3$ respectively, so that the darker lines belong to the negative subsets and the brighter lines belong to the positive subsets. Note that all the lines are thin, even for higher values of n. The structuring-function used is flat with the shape of a 2x2 square.

As in the binary case, the reconstruction process is performed iteratively from pyramidal level $n+1$ to the higher-resolution level n with the "addition" of the information of the positive and negative skeleton functions:

$$\begin{aligned} f_n = \{\{[f_{n+1} \oplus n \cdot g \\ - s_n^-] \ominus 2n \cdot g \\ + s_n^+\} \oplus n \cdot g\} \bullet n \cdot g \end{aligned} \quad (8)$$

If $f(x, y)$ is spatially-bounded, i.e., if there exists a positive real M such that $f(x, y) = 0$ for $x^2 + y^2 > M$, then there exists a natural N such that $f_n \equiv 0$, $\forall n \geq N$. This means that all the information is retained in the Skeleton Functions of orders less or equal to $N - 1$, and that an error-free reconstruction from those functions can be obtained.

Partial reconstructions can be obtained as well, by stopping the reconstruction process at any level $k > 0$.

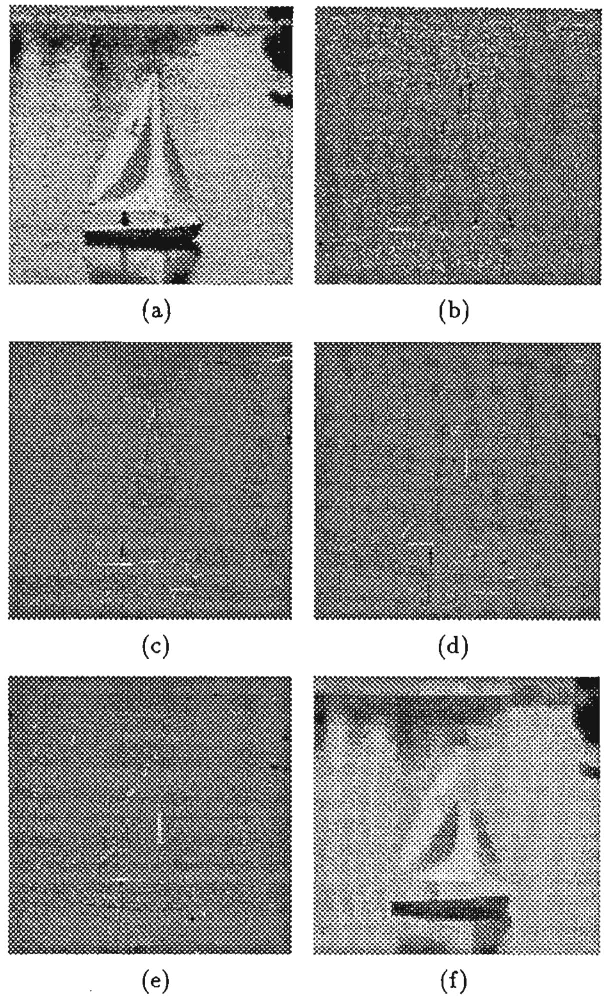


Fig. 6: The Two-Sided Skeleton of grayscale images. (a) The same grayscale image as in Fig. 2(a), (b)-(e) composition of its Positive and Negative Skeleton Functions of order 0 to 3, respectively, (d) the fourth level of the ASF-based pyramid.

III. APPLICATIONS

Some of the application areas, in which the Two-Sided Skeleton can be applied, are examined bellow.

A. Multi-Resolution Analysis

The positive and negative skeleton subsets (in the case of binary images) or functions (in the case of grayscale images) constitute an error-free "band-pass" pyramid, when the related "low-pass" pyramid is the ASF-based pyramid defined in (1) or (2). In this context, the concepts "band-pass" and "low-pass" relate not to frequencies, but to size. But like frequency "band-pass" pyramids, the Two-Sided Skeleton contains at its lower levels (i.e., its subsets - or functions - of lower orders) the finest details of the image, and at its higher levels the largest components of the image.

The information contained at some level n of this "band-pass" pyramid may be viewed as related to those features that belong to *resolution level n*, but not to the lower *resolution level* $n+1$. In this sense, a multi-resolution analysis (and/or processing) can be performed based on the Two-Sided Skeleton subsets (or functions).

As pointed out in section I, the Two-Sided Skeleton is preferable to the "band-pass" pyramid presented in [1] (Fig. 2(b)) and the to *thinned* version of the Feature-Width Pyramid presented in [2, 3], because its implicit thinning process is error-free. This prevents the pyramidal levels to contain spurious features, and therefore provides a more consistent representation.

B. Robust Representation

Although it is not a robust representation, the ordinary Skeleton has some degree of insensitivity to *positive noise*, which is noise that contaminates only the *background* of a binary image, or that only *increments* the values of a grayscale image. However, *negative* *noise*, which contaminates the *foreground* of a binary image or *decrements* the values of a grayscale image, often alters completely the ordinary Skeleton of an image. Fig. 7(a)-(d) illustrate the behavior of the ordinary Skeleton of a binary image in presence of negative noise: Fig. 7(a) is the original binary image and Fig. 7(b) is the same image with 1% negative binary noise. Fig. 7(c) and Fig. 7(d) are the ordinary Skeleton of the images in Fig. 7(a) and Fig. 7(b), respectively. Note how the shape of the skeleton changes.

The Two-Sided Skeleton has the same degree of insensitivity to both positive and negative noise. Since in many applications the corrupting noise has both positive *and* negative components, the Two-Sided Skeleton should be preferred for working under noisy conditions. Fig. 7(e-f) demonstrate the behavior of the Two-Sided Skeleton in the presence of noise. Fig. 7(e) and Fig. 7(f) are the Two-Sided Skeleton of Fig. 7(a) and Fig. 7(b), respectively, where the black points refer to the negative skeleton and the white points to the positive skeleton. The structuring-element used in the simulation was *Rhombus* (the origin and its 4-pixel neighborhood).

C. Compression and Progressive Transmition

The Two-Sided Skeleton subsets (or functions) provide a compact representation of the original image, which permits also efficient Coding of the image.

In the grayscale case, compression may be achieved by techniques similar to these applied to other "band-pass" pyramids. Some of those techniques are:

- Allowing a different quantization error at each resolution level. The higher the resolution level, the greater the permitted quantization error. The highest resolution level(s) may sometimes be totally discarded. (The highest resolution levels are related to the lowest function orders).
- Applying different coding methods at different groups of resolution levels.

For instance, the Two-Sided Skeleton can replace the thinned version of the Feature-Width Pyramid in the coding method proposed in [2] and [3]. This is expected to improve the quality of the reconstructed image because there is no loss of information in the Two-Sided Skeleton implicit thinning process. After the Two-Sided Skeleton is obtained, the coding is done as in [2] of [3]: the *highest resolution levels* are decomposed by a directional morphological filter bank into directional images, which are scanned in the respective direction and have their run-lengths coded. The remaining coarse image is coded by a vector quantization scheme.

The interest in using a *morphological* pyramid, such as the Two-Sided Skeleton for image coding, is that it represents image edges in a more efficient way than linear pyramids. Since the ordinary Skeleton represent explicitly the edges of *positive elements* only, it is less suitable than two-sided representations for efficient coding.

In the binary case, the Two-Sided Skeleton usually achieves compression rates similar to those of the ordinary Morphological Skeleton, even though there are two Skeleton subsets at each step n, instead of only

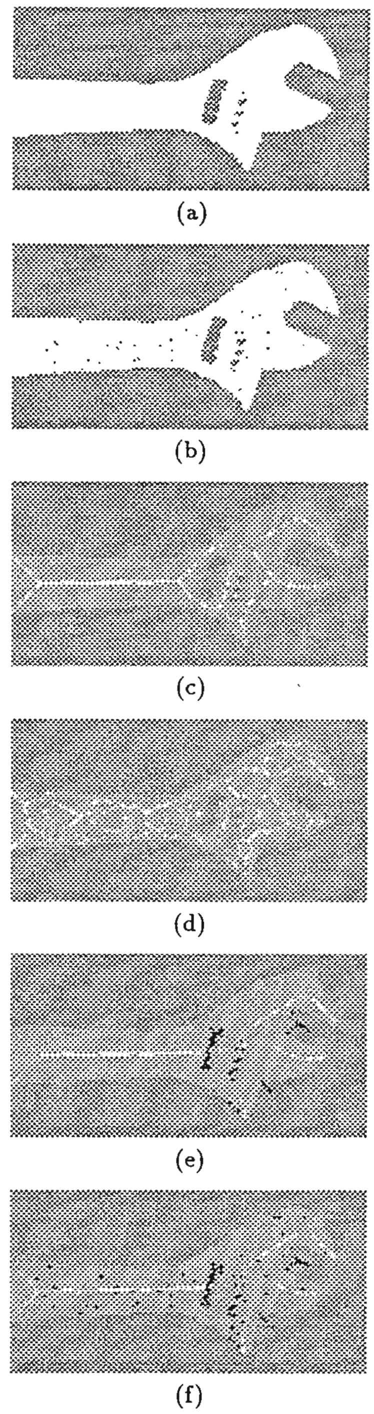


Fig. 7: The behavior of the ordinary and the Two-Sided Skeletons under noisy conditions: (a) A grayscale image, (b) The same image, but with 1% binary negative noise, (c) the ordinary Skeleton of (a), (d) the ordinary Skeleton of (b), (e) the Two-Sided Skeleton of (a), (f) the Two-Sided Skeleton of (b).

one as in the ordinary Skeleton. This is because there are usually less representation points in the Two-Sided Skeleton than in the ordinary Skeleton, as was demonstrated in Fig. 4.

In either case, the Two-Sided Skeleton is also suitable for Progressive Transmition (from the highest subset orders to the lowest ones), since the reconstruction is performed on a level-by-level basis and since each level "adds" more details.

D. Image Decomposition

Every Two-Sided-Skeleton point is accompanied by two parameters: size (the order of the skeleton subset or function) and side of the Pattern Spectrum (whether it belongs to a positive or to a negative subset or function). By selecting appropriate Two-Sided-Skeleton points, one can obtain decompositions of the original image, according to those parameters.

E. Pattern Recognition

Each Two-Sided Skeleton point is the center of a dilated version of the structuring-element. If the structuring-element is convex, the dilated versions are also scaled versions of it. Thus, it is possible to search for a certain convex shape in a given image by analyzing the Two-Sided Skeleton of that image, when the structuring-element used in the calculation is the convex shape to be found. Such method, *using the ordinary Skeleton*, is expected to detect positive elements only.

IV. CONCLUSIONS

A new morphological representation structure, the Two-Sided Skeleton, has been defined, both for binary and grayscale images, and its applications were discussed. It was shown that the new structure is able to represent image details in a more meaningful way than the ordinary Morphological Skeleton, since it considers both the positive and the negative features of the image. The Two-Sided Skeleton characteristics as a multi-resolution pyramid, and its advantages over two other similar morphological multi-resolution representations, were also pointed out.

APPENDIX

Proof of the reconstruction relation in equation (5).

First we notice that for all n:

$$X_{n+1} \bullet (n+1)B = X_{n+1} \qquad (A.1)$$

$$X_{n+1} \bullet nB = X_{n+1} \qquad (A.2)$$

Relation (A.1) is a direct consequence of the definition for $\{X_n\}$ in (2), and (A.2) is obtained from (A.1).

Using (A.2), and since $A \ominus B - C \oplus B^s = (A - C) \ominus B$, $\forall A, B, C$, we get:

$$\begin{aligned} X_{n+1} - (S_n^- \oplus nB^s) &= \\ &= [(X_{n+1} \oplus nB) \ominus nB] - (S_n^- \oplus nB^s) \\ &= [(X_{n+1} \oplus nB) - S_n^-] \ominus nB \qquad (A.3) \end{aligned}$$

Also $S_n^- = (X_{n+1} \oplus nB) - (X_n \oplus nB)$ (by definition), and $A - (A - B) = A \cap B$, $\forall A, B$. Therefore:

$$\begin{aligned} [(X_{n+1} \oplus nB - S_n^-] \ominus nB &= \\ &= [(X_{n+1} \oplus nB) \cap (X_n \oplus nB)] \ominus nB \\ &= [(X_{n+1} \bullet nB) \cap (X_n \bullet nB)] \qquad (A.4) \end{aligned}$$

From (A.1), (A.2), (A.3) and (A.4), we obtain:

$$X_{n+1} - (S_n^- \oplus nB^s) = X_{n+1} \cap X_n \qquad (A.5)$$

Performing opening and "adding" now the information in S_n^+, it follows:

$$\begin{aligned} [(X_{n+1} \cap X_n) \circ nB] \cup (S_n^+ \oplus nB) &= \\ &= [(X_{n+1} \cap X_n) \ominus nB \cup S_n^+] \oplus nB \\ &= [(X_{n+1} \ominus nB) \cap (X_n \ominus nB) \cup S_n^+] \oplus nB \end{aligned}$$

Using $S_n^+ = (X_n \ominus nB) - (X_{n+1} \ominus nB)$ (by definition) and noting that $(A \cap B) \cup (A - B) = A$, $\forall A, B$, we obtain:

$$\begin{aligned} [(X_{n+1} \ominus nB) \cap (X_n \ominus nB) \cup S_n^+] \oplus nB &= \\ &= [(X_{n+1} \ominus nB) \cap (X_n \ominus nB) \cup \\ &\quad \cup (X_n \ominus nB) - (X_{n+1} \ominus nB)] \oplus nB \\ &= (X \ominus nB) \oplus nB = X_n \circ nB \qquad (A.6) \end{aligned}$$

Finally we use the fact that the opening-closing operation is idempotent, and that $X_n = X_{n-1} \circ nB \bullet nB$, to state that:

$$(X_n \circ nB) \bullet nB = X_n \qquad (A.7)$$

The reconstruction relation in (5) is then obtained from (A.5), (A.6) and (A.7).

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