Codebook Index Assignment by an Approximate Solution of the Traveling Salesman Problem

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ABSTRACT

Index Assignment (IA) is a process of indexing the vectors in a codebook (vector quantizer) for the purpose of reducing the distortion caused by transmission over a channel with errors. Achieving an optimal IA is difficult since it is an NP-complete problem. Common approximate solutions to the IA problem consist of iterative algorithms which gradually reduce a distortion measure, till reaching a local minimum.

In this paper we propose a new method for IA which is based on an approximate solution of the Traveling Salesman Problem (TSP). The proposed method has a low complexity dependent only on the number of vectors in the codebook. It results in a distortion not much larger than that achieved by the "natural" ordering obtained from the LBG-splitting codebook design algorithm, thus enabling a fast and simple IA when the "natural" IA is not given.

1 INTRODUCTION

The indices assigned to codevectors in a codebook are of importance when these indices are transmitted via a channel with errors. When an error occurs, the received index is different from the transmitted one and the receiver extracts the wrong vector from the codebook. A proper IA can reduce the distortion by trying to avoid situations in which a vector is replaced by a distant vector. If a binary-symmetric channel (BSC) is used, the indices of the original and replacing vectors typically have a Hamming distance of 1 between them.

The IA problem is a Quadratic Assignment problem which Sahni and Gonzalez [1] not only have shown to be NP-complete, but belonging to the hardest type of these problems: finding an ε-approximate solution to it is also NP-complete. Problems of size greater than 15 are considered difficult to solve [2].

Known sub-optimal solutions for the IA problem include a Binary Switching Algorithm (BSA) presented by Zeger and Gersho [3], a Simulated Annealing Algorithm (SAA) introduced by Farvardin [4], and a Linearity Increasing Swap Algorithm (LISA) suggested by Knagenhjelm [5]. These methods are based on decreasing a defined distortion function by switching indices between codevectors. Such an approach results in iterative algorithms which gradually reduce the distortion, till reaching a local minimum.

Cheng and Kingsbury mentioned in [6] that the "natural" IA generated by the vector quantization LBG-splitting algorithm [7] tends to be significantly superior to random IA. According to our simulation results, the BSA initialized by a random assignment typically reaches the level of distortion of the "natural" assignment only after many iterations. If the BSA is initialized by the "natural" assignment, its improvement is not significant.

The paper is organized as follows. In Section 2 we describe the IA problem. The travelling salesman problem and its solution by the farthest insertion algorithm appear in Section 3. In Section 4 we present the proposed IA method. In Section 5 we compare the performance of the proposed method to those of the "natural" IA and a random IA. Concluding remarks are made in Section 6.

2 THE INDEX ASSIGNMENT PROBLEM

A codebook of size N is composed of N vectors (or codevectors) which are denoted by y_i , $i = 0,1,\ldots,N$. A measure for the average distortion caused by transmission over a channel with errors is:

$$D_C = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} p_i p_{j|i} d(y_i, y_j) \tag{1}$$

where p_i is the probability of transmitting the codevector y_i, $p_{j|i}$ is the probability of sending the index i and receiving the index j due to channel errors and $d(y_i, y_j)$ is the distance between the vector y_i and the vector y_j.

In the case of a BSC with a small bit error probability of q we can approximate $p_{j|i}$ by:

$$p_{j|i} \approx \begin{cases} q & ham(i,j) = 1 \\ 0 & ham(i,j) > 1 \end{cases} \quad (2)$$

where $ham(i,j)$ is the Hamming distance between i and j.

p_i is dependent on the distribution of the data vectors represented by the codevector y_i. If this distribution is not known, it is common to assume that it is uniform: $p_i = 1/N$.

Under these assumptions the average distortion becomes:

$$D_C = \frac{q}{N}\sum_{i=1}^{N}\sum_{j|ham(i,j)=1} d(y_i, y_j) \quad (3)$$

It is this distortion we're trying to minimize here by the process of index assignment (IA).

3 THE TRAVELING SALESMAN PROBLEM

The TSP is the problem of finding the shortest closed tour passing only once through each one of N points in a K dimensional space [8]. This too is an NP-complete problem, but one for which extensive research has yielded many approximate solutions that give very good results with relatively low complexity.

One of the simplest solutions is the Farthest Insertion Algorithm (FIA) [8]. This algorithm works for TSPs with a distance measure that satisfies the triangle inequality. The FIA's steps are:

Step 1: Start with a partial tour consisting of a single point i.

Step 2: If the current partial tour T does not include all the points, find the point k not on T with the maximum distance from T, i.e., with the maximum distance from the closest point j on T (Fig. 1a).

Step 3: Add k to the partial tour T by inserting it between j and one of the two tour neighbors of j. The neighbor is selected as to minimize the added tour length (Fig. 1b).

Note: In the first two iterations j has less than two neighbors. In these iterations add k to T anywhere.

Step 4: Repeat steps 1-3 N times, till all the points are added to the tour T.

Step 5: Repeat steps 1-4 N times, each time beginning with a different point i. From the N generated tours select the shortest one.

The FIA has complexity of $O(N^3)$. It is a heuristic approach that doesn't have analytic bounds on its performance. There have been specially generated instances of the TSP for which the FIA tour was nearly 50% longer than the optimal one, but for typical instances it is within 5% of the optimal tour [8].

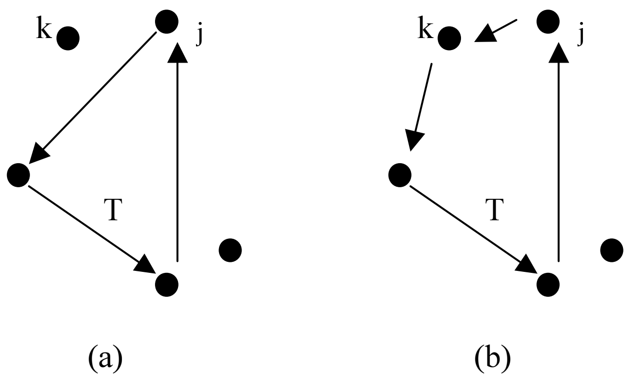

Fig. 1: (a) Selecting the point to be added to the tour. (b) Adding the point to the tour.

A crude but useful method for evaluating the quality of the FIA tour, for a given set of N points, is comparing its length to the length of the Minimum Spanning Tree (MST) of the points. A MST is a connected graph passing through all the points and having a minimum sum of the arcs' lengths. The length of the MST is a lower bound on the length of the optimal tour for the TSP, while twice its length is an upper bound on the length of the optimal TSP tour. The MST can be found in $O(N^2)$ [8].

The FIA results could be improved by applying another heuristic approach: using the dual Nearest Insertion Algorithm (NIA) [8], and selecting the best tour from the results of both algorithms. The average performance of the NIA for typical instances of the TSP is similar to that of the FIA. The complexity of the NIA is also $O(N^3)$. Using both algorithms can give a somewhat better tour in some instances at the expense of doubling the complexity.

4 PROPOSED IA METHOD

The solution proposed in this paper is for cases where the "natural" IA is not given, either because the codebook was not designed by the user or the codebook design was performed by means other than the LBG algorithm. The proposed method is to use an approximate solution of the TSP (Section 3). In the cases we tested the resulting distortion was only slightly higher than that obtained by the "natural" index assignment. This IA can then be used as is, or further improved by any of the iterative IA methods mentioned in Section 1.

It is shown in [8] that by solving the TSP one can solve the "Computer Wiring Problem" which in our case is the problem of ordering (indexing) the codevectors in such a way that the sum of the distances along the tour is minimal:

$$\sum_{i=1}^{N-1} d(y_i, y_{i+1}) \quad \rightarrow \quad \min \tag{4}$$

The effectiveness of the proposed method lies in the fact that an ordering that minimizes $\sum_{i=1}^{N-1} d(y_i, y_{i+1})$ tends to minimize $\sum_{i=1}^{N} \sum_{j|ham(i,j)=1} d(y_i, y_j)$.

The codeword given to each codevector is the binary representation of the codevector's index in the ordering. This ordering usually has two properties regarding codevectors with codewords having a Hamming distance of 1 between them: Minimal distance between codevectors with codewords different only in the LSB and a monotonically increasing distance between codevectors as the bit by which the codewords differ moves towards the MSB. The "natural" ordering of the LBG gets its strength from the same properties, and therefore the FIA is expected to give results similar to those of the LBG-based ordering.

As previously mentioned, the complexity of the FIA is of $O(N^3)$. I.e., it is dependent only on the size of the codebook and not on the distribution of its codevectors. The actual run time is similar for different codebooks of the same size and is relatively short. The run time of a non-optimized version of the algorithm is about 2% of that of the Binary Switching Algorithm (BSA) applied to the same codebook.

5 SIMULATION RESULTS

We tested this method on codebooks of size 2^8 and 2^9 from [9]. Each codebook contains either 4-dimensional or 6-dimensional codevectors produced by the LBG algorithm from Line Spectral Frequency (LSF) vectors of speech [10]. The distance used was the Euclidean distance. The resulting distortion was measured as the mean of the squared distances between codevectors having codewords with a Hamming distance of 1 between them. For these codebooks the "natural" IAs are better by 4.8dB, on average, than the average distortion obtained for 40 random IA's. The proposed method results in IAs with a distortion higher than that of the "natural" IAs by less than 1.2dB. That is, the TSP solution resulted in an improvement of at least 3.6dB over random assignments. When the BSA was applied to any of the mentioned IA's, they all reached a similar level of distortion, which at best is 0.6dB below that of the initial "natural" IA.

Fig. 2 shows the convergence of the BSA for a codebook of size 2^8 with 6-dimensional vectors. The BSA was initialized by three different IAs: the "natural" IA, a random IA and the TSP IA. It can be seen that the initial TSP IA has a distortion much smaller than that of the initial random IA and only slightly bigger than that of the initial "natural" IA. It is also evident that the BSA doesn't reduce much the distortion of both the initial "natural" IA and the initial TSP IA.

Fig. 3 shows the total distortion of the system (quantizer and channel) for different Bit Error Rates (BER). These results were obtained by encoding and decoding a speech segment using a VQ with 2^8 vectors of length 10. The vectors were generated by joining the 4-dimensional and 6-dimensional vectors mentioned above [11]. The distortion is measured as the mean of the squared distances between the original vectors and the restored ones. The results are for the initial "natural" IA, the initial TSP IA, the initial random IA and for all these IAs after using the BSA on them. The results after the BSA are very similar and coincide in Fig. 3.

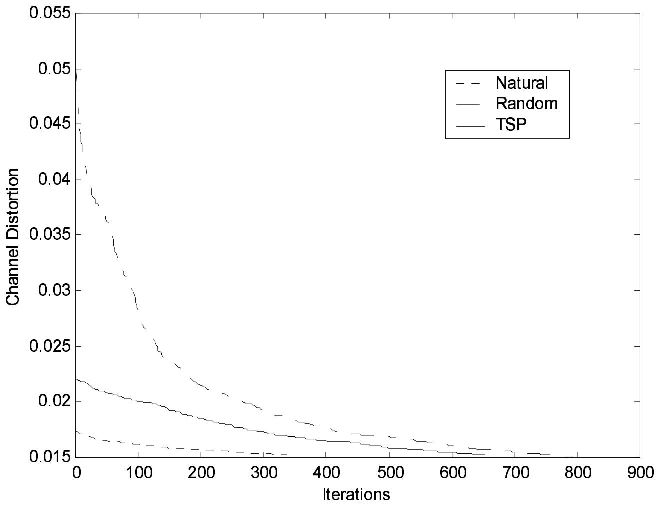

Fig. 2: Convergence of the BSA for three different initial IAs. The top line is the random IA, the middle line is the TSP IA and the bottom one is the "natural" IA. The distortion is measured as the mean of the squared distances between vectors with indices that have a Hamming distance of 1 between them.

We should note that for LSF vectors the Log Spectral Distance (LSD) is more appropriate than the Euclidean distance. The proposed method should work also for the LSD since it satisfies the triangle inequality [9].

6 CONCLUSION

The proposed algorithm, which is based on an approximate solution of the TSP, is a simple and fast method for generating a good IA when a "natural" IA (as obtained from the LBG-splitting algorithm) is not given. The performance of this IA is substantially better than that of an average random IA and its computational load is much smaller than that of the BSA or other iterative IA methods applied to an initial random IA.

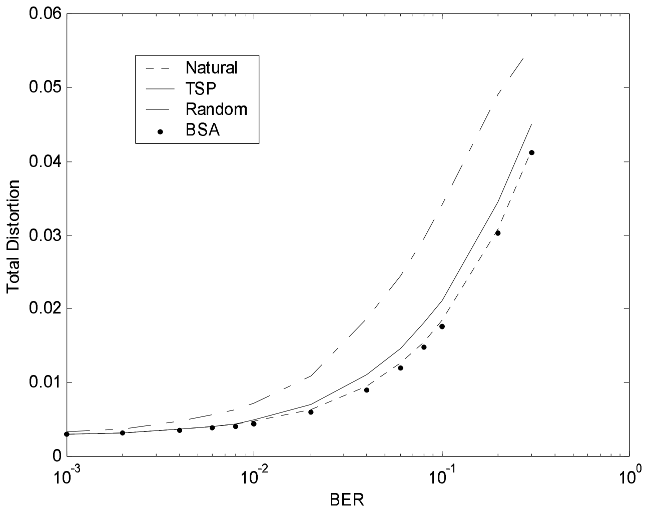

Fig. 3: The total distortion of the system (quantizer and channel) as a function of BER for four different IAs. The top line is the random IA, the middle line is the TSP IA, The bottom line is the "natural" IA and the dots are the IAs after BSA. Distortion is measured as the mean of the squared distance between the original vectors and the restored ones.

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