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DIGITAL WIENER FILTERS FOR MULTIECHOED SIGNAL DECOMPOSITION

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Abstract

The paper presents a study of digital Wiener filters designed for decomposition of multiechoed signals. Design methods for two important situations: input signals immersed in noise and unknown wavelet shape, are further developed.

For noisy input signals, care must be taken in designing the filter in order to avoid excessive output noise which will mask the desired signal. It is shown that by varying the noise weighting factor in the cost function a trade-off between signal resolution and noise error is obtained. An optimal weighting factor for many practical situations is derived and verified in simulations. It is also shown that output noise can be controlled by varying the width of the output pulse.

The paper also considers the important case of an unknown wavelet shape. A known approach for this situation is extended to enable the decomposition by Wiener filters of certain classes of signals.

Introduction

In many physical situations, we encounter signals composed of a basic wavelet and replicas of this wavelet, immersed in noise. Such waveforms are common in radar, sonar, seismology, acoustic recording, brain waves, etc. It is often required to determine the number, amplitude and relative occurance of various echoes.

In a comparative study of existing techniques of signal decomposition [1] we have found that in situations of poor SNR and unknown wavelet shape the Wiener decomposition filter is particularly suitable. The digital Wiener decomposition filter shapes the basic wavelet, s_t, into a narrow output pulse - g_t. The response of the filter to an input signal composed of the basic wavelet and overlapping echoes, x_t, is therefore a series of narrow non-overlapping pulses:

(1a) $$x_t = s_t + \sum_{i=1}^{k} a_i \cdot s_{t-T_i}$$

(1b) $$y_t = g_t + \sum_{i=1}^{k} a_i \cdot g_{t-T_i}$$

s_t - Basic Wavelet
a_i - amplitude of i'th echo
T_i - time of occurance of i'th echo
k - number of echoes
x_t - composite input signal
g_t - desired output
y_t - output signal

Fig. 1 demonstrates the decomposition of the signal appearing in Fig. 1b, which is an echoed version of the wavelet in Fig. 1a. The outputs of three Wiener filters, designed for desired outputs of unit impulse and Gaussian pulses with σ = 2.4 and 4.5 are shown in Fig. 1c. The narrow pulses of the output provide the information about the times of arrival and amplitudes of the echoes.

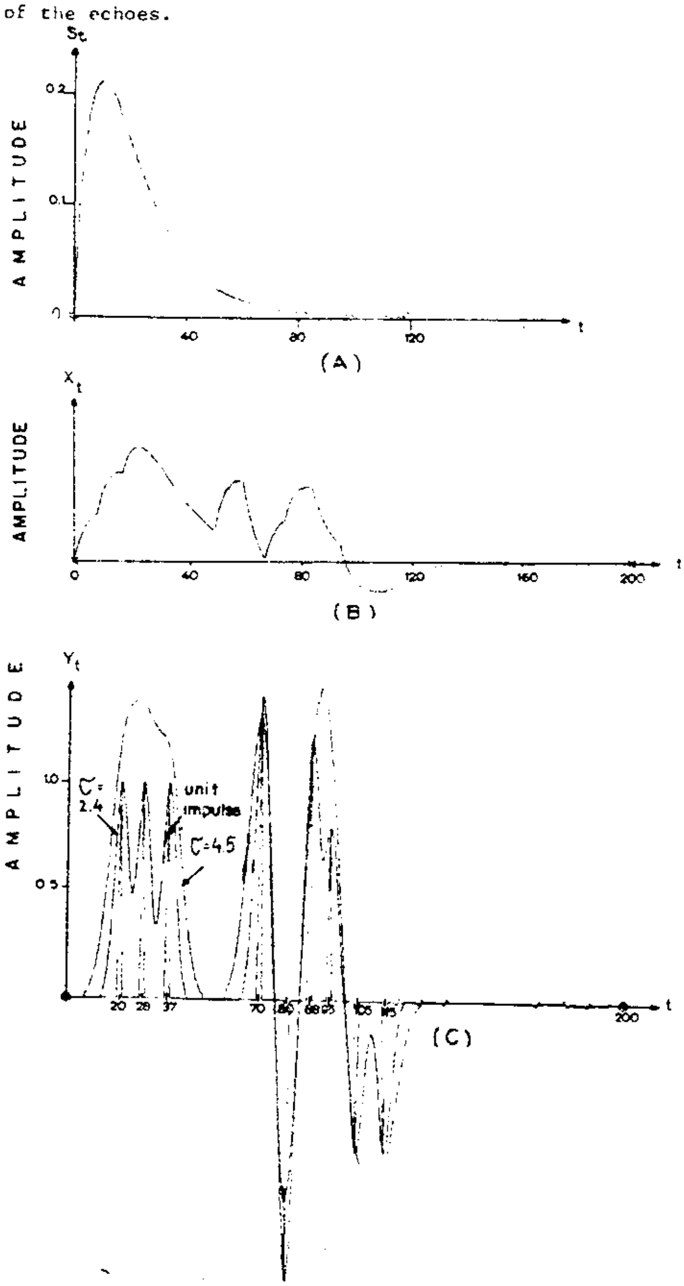

Figure 1.

General description of Wiener Shaping Filters

The Wiener shaping filter is a linear, time-invariant, finite impulse response filter (FIR) designed to shape the basic input wavelet into a narrow pulse, using the least mean square error approach. For an input signal composed of a wavelet, s_t and noise, u_t, the mean square error between the actual output and the desired output is given by:

$$(2)\qquad I = E\{x_t * h_t - g_t\}^2 = \sum_{t=0}^{M+N-2} \left(g_t - \sum_{\ell=0}^{N-1} h_\ell \cdot s_{t-\ell}\right)^2 + E\{v_t^2\}$$

x_t - input signal; $x_t = s_t + u_t, t = 0,1,\ldots,M-1$.

u_t - input stationary noise, uncorrelated with s_t.

h_t - linear time invariant FIR filter, $t = 0,1,\ldots,N-1$.

v_t - output noise: $v_t = u_t * h_t$

The Wiener shaping filter minimizes the error I [2]. In [3] it is suggested to generalize the cost function (2) by weighting its two terms

$$(3a)\qquad I = E_s + w \cdot E\{v_t^2\}$$

where

$$(3b)\qquad E_s = \sum_{t=0}^{M+N-2} \left(g_t - \sum_{\ell=0}^{N-1} h_\ell \cdot s_{t-\ell}\right)^2$$

E_s is the signal error and **w** is a positive weighting factor, which will be discussed in the sequel.

The Wiener shaping filter coefficients are derived by minimization of (3). For real signals the minimization leads to the following set of equations [3]:

$$(4a)\qquad \begin{bmatrix} R_0 & R_1 & R_2 & R_{N-1} \\ R_1 & R_0 & R_1 & R_{N-2} \\ R_{N-1} & R_{N-2} & R_{N-3} & R_0 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} \phi_{gs} & (0) \\ \phi_{gs} & (1) \\ \phi_{gs} & (N-1) \end{bmatrix}$$

$$(4b)\qquad R_i = \phi_{ss}(i) + w \cdot \phi_{uu}(i)$$

With ϕ_{ss} and ϕ_{uu} being the autocorrelation functions of the input wavelet and noise, respectively, and ϕ_{gs} the cross-correlation between the wavelet and the desired output.

In the noiseless case there are three design parameters which may reduce the error energy of the Wiener filter (obtained from (4) with $\phi_{uu} = 0$):(a) Filter's memory duration (its increase asympototically reduces I). (b) Desired output delay (optimal delay depends on wavelet properties). (c) Desired output shape. In decomposition problems it is accepted to use the Gausian pulse as the desired output:

$$(5)\qquad g_t = \begin{cases} (2\cdot\pi\cdot\sigma)^{-0.5} \cdot \exp\left\{\dfrac{(t-\tau)^2}{2\sigma^2}\right\}, & \tau-2\tau \le \tau \le \tau+2\sigma \\ 0 & \text{else} \end{cases}$$

The design parameter is thus σ . It can be shown [4], [1], that if σ is chosen to satisfy:

$$(6)\qquad \sigma \le 0.25\ T\ \text{min}$$

where T min is the time interval between the two pulses, then the two pulses can be satisfactorily distinguished.

The design considerations for these parameters in the noiseless case are described elsewhere ([1], [3]).

Decomposition of Signals Immersed in Noise

Usually the composite signal includes noise which interferes in the detection of the echoes as well as introduces errors in the estimation of their amplitudes and delays. This situation is hardly treated in the literature except for the "classical" approach based on (2) (w = 1).

For high SNR_{in} or for unknown noise statistics the output noise can be reduced by increasing the desired output's width σ. (SNR is defined as the ratio between maximal signal amplitude and r.m.s. value of the noise). The filter coefficients are then solved from (4) with w = 0.

As a measure for the output noise we may use the sum of squares of the filter's coefficients. For white and uncorrelated input noise and high resolution filter, SNR_{out} is inversely proportional to this measure:

$$(7)\qquad SNR_{out} = SNR_{in} \cdot \left(\sum_{i=0}^{N-1} h_i^2\right)^{-0.5}$$

Increasing the output pulse width increases SNR_{out} but decreases resolution. It is possible to define a measure of performance as a combination of resolution and output noise measure and to look for σ which optimizes it [4]. However, in most of the simulations carried out in [1] we found that it is both effective and convenient to choose the pulse width according to σ = 0.25·T min. Fig. 2 demonstrates the effect of white Gausian noise on the Wiener filter output. Noise is added to the input described in Fig. 1b to form the noisy input (Fig. 2a). The actual Wiener filter's outputs are presented in Fig. 2b-Fig. 2d. Wiener filters are designed for desired outputs of a unit sample (Fig. 2b) and Gausian pulses with σ = 2.4 (Fig. 2c) and σ = 4.5 (Fig. 2d). The compromise between resolution and output noise achieved by σ = 0.25·T min≈2.4 is self evident.

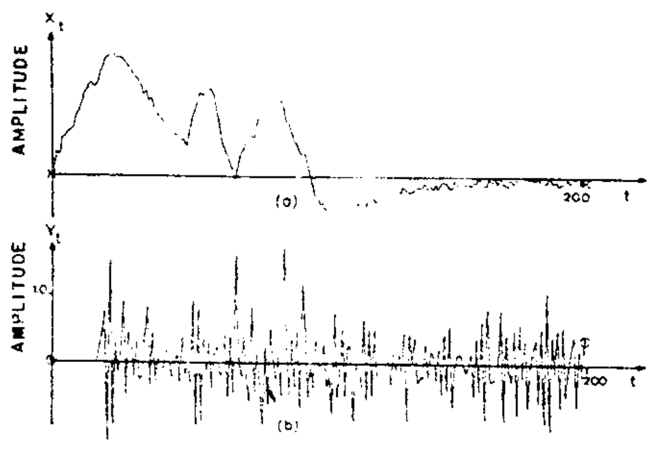

Figure 2.

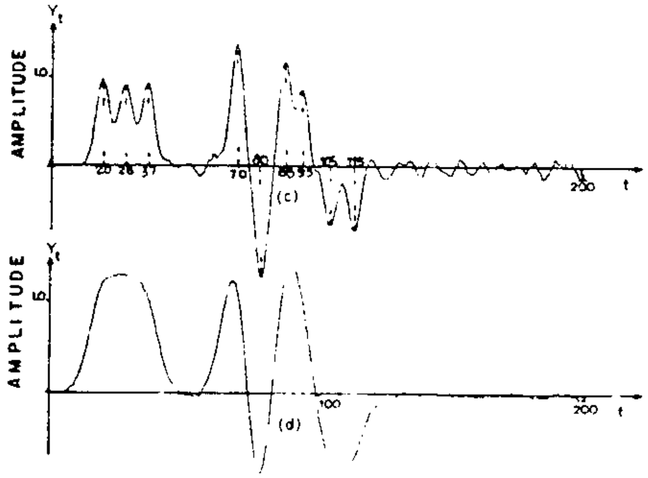

Figure 2 (Continued)

Improving the Decomposition of Noisy Signals by Weighting Output Noise

When the input noise statistics are known it is possible to improve Wiener filter's performance by a careful selection of w in (3a), following the selection of σ as described above.

The Effect of w on Filter Coefficients and Output.

The first term of the cost function I in (3a) is the error between the signal output - $s_t * h_t$ and the desired output. The second term is the weighted output noise power. Increasing w increases the weight of the output noise. For stationary white noise (3a) becomes:

$$(8) \qquad I = E_s + w \cdot E\{n_t^2\} \cdot \sum_{i=0}^{N-1} h_i^2$$

From (8) it appears that the increase of w will decrease the sum of squares of the coefficients. Fig.3 presents the dependence of $\sum h_i^2$ on w as computed after minimizing (3a). The basic wavelet was recorded in an impact test of an iron rod. The input noise is white and Gausian (at SNR_{in} = 19). The desired outputs are unit impulse and Gausian pulses with σ = 2.4, 4.5. The three graphs in Fig. 3 show clearly that $\sum h_i^2$ decreases as w increases. It then can be shown [1] that the signal error energy E_s increases with w.

Thus, increasing w reduces output noise at the expense of greater error between signal output and the desired output.

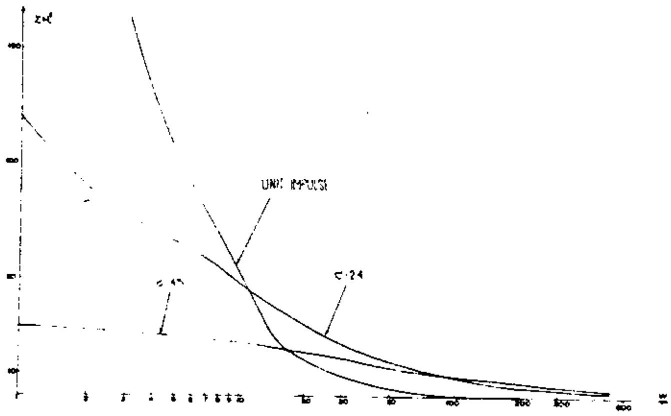

Figure 3

Optimal Selection of w.

We shall define the optimal w as the one which minimizes

$$(9) \qquad I^* = \sum_{t=0}^{L-1} (y_t - g_t^*)^2$$

I^* - error energy at the output of the decomposition filter.

L - output duration

y_t - actual Wiener filter output

g_t^* - desired output for the specific input (wavelet and echoes).

The error function (9) can be developed [1] into:

$$(10a) \qquad I^* = \sum_{t=0}^{L-1} (P_t * (s_t * h_t - g_t))^2 + L \cdot E\{v_t^2\}$$

Where:

$$(10b) \qquad P_t = \delta_t + \sum_{i=1}^{K} a_i \cdot \delta_{t-T_i}$$

P_t is the reflection series of the input signal as defined in (10 b).

The optimal selection of w results in a Wiener filter calculated from (4) which minimizes I^* in (10a). It is difficult to derive a general expression for the optimal w. However, in most practical situations we may assume that $(s_t * h_t - g_t)$ is narrower than the time delay between two echoes, because g_t is narrower than the time delay. Under this assumption (10a) becomes:

$$(11) \qquad I^* = (1+\sum_{i=1}^{K} a_i^2) \cdot \{E_s + L \cdot (1+\sum_{i=1}^{K} a_i^2)^{-1} \cdot E\{v_t^2\}\}$$

According to its definition, the Wiener filter calculated from (4) minimizes the cost function I of (3a), i.e.:

$$(12) \qquad I = c \cdot \{E_s + w \cdot E\{v_t^2\}\}$$

c - arbitrary constant

Hence, from (11) and (12) we find that the optimal selection of w is:

$$(13) \qquad W_{opt} = L \cdot (1+\sum_{i=1}^{K} a_i^2)^{-1}$$

Simulations carried out in [1] confirm that the optimal w calculated from (13) is a good starting point even in situations where our assumption is no longer valid. Fig. 4 shows the output of Wiener filters designed for various values of w (σ=2.4), for the input signal in Fig. 4a. Calculation of the error I^* defined in (9) shows the optimal w is approximately 8 as calculated in this case from (13).

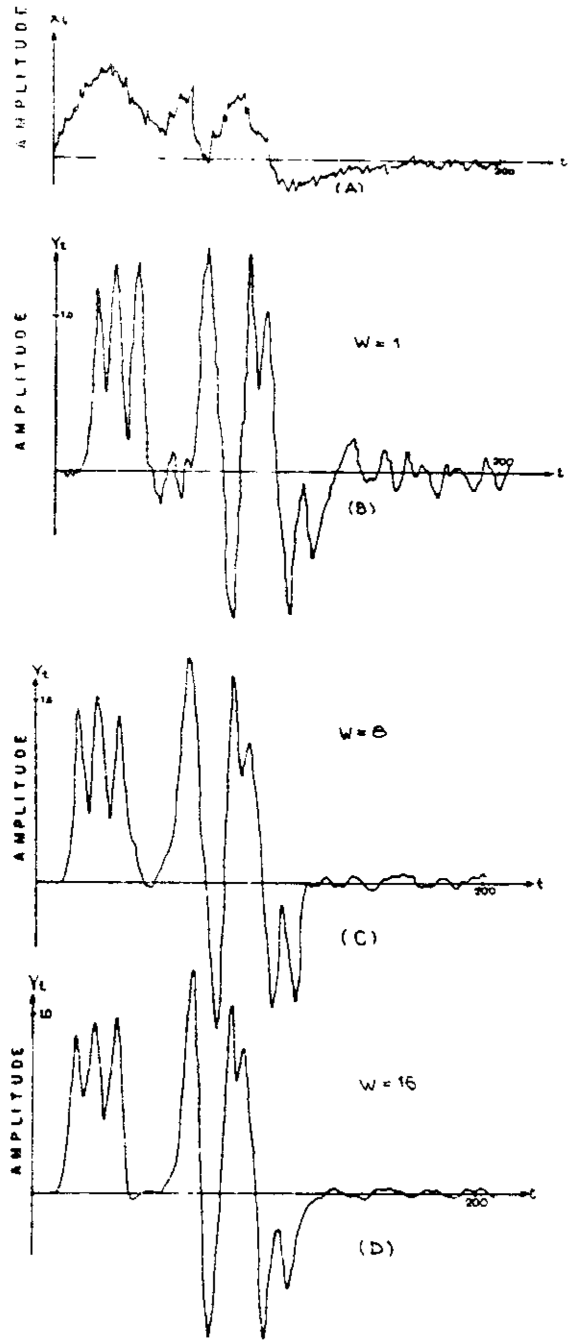

Figure 4

Wiener Decomposition Filters for Unknown Wavelets

Using Wiener decomposition filters requires information on the wavelet shape. This information is not always available. In order to design the Wiener filter from (4) we have to know ϕ_{ss} and ϕ_{gs}. (We shall treat in this section the noiseless case only, so that $\phi_{uu}(i)=0$). Sometimes it is possible to estimate this correlation for unknown wavelets.

Estimation of Wavelet's Autocorrelation

In certain situations it is possible to use the autocorrelation of the input signal in a limited time interval. It can be shown that:

(14) $\phi_{xx}(j) = \phi_{ss}(j) * \phi_{pp}(j)$

p_t is the reflection series of input signal, as defined in (10a).

In the following two cases such an approximation is possible.

(a) The reflection series p_t is uncorrelated. In this case [5] we have:

(15) $\phi_{xx}(j) = \phi_{pp}(0) \cdot \phi_{ss}(j)$

(b) It is sufficient that:

(16) $\phi_{xx}(j) \approx \phi_{pp}(0) \cdot \phi_{ss}(j)$, $j = 0, \ldots N-1$

where N - is the filter's memory duration.

The approximation (16) depends on the functions ϕ_{ss} and ϕ_{pp}. Specifically, (16) is true if:

(17) $\phi_{pp}(0) \cdot \phi_{ss}(j) >> \sum \phi_{pp}(t_k) \cdot \phi_{ss}(j-t_k)$ $j=0,1,\ldots$

where t_k are the values of t for which $\phi_{pp}(t)$ is not zero.

The condition (17) implies that the filter's memory duration, N, is shorter than the minimal delay between the echoes, T_{min} and that ϕ_{ss} is relatively narrow.

Estimation of the Cross Correlation Between the Wavelet and the Desired Output- ϕ_{gs}

For unknown wavelets ϕ_{gs} can be calculated in two ways:

(a) By arbitrarily selecting ϕ_{gs} we can control the desired output's width which is satisfactory for decomposition problems [6].

(b) The cross correlation ϕ_{gx} may sometimes be used instead of ϕ_{gs}. This approximation holds if only the basic wavelet is present in the time interval between zero and the end of the desired output (for Gausian output pulse (5) the interval is $[0, \tau + 2\sigma]$).

Figure 5a describes a multiechoed signal with an unknown basic wavelet. Fig. 5b presents the signal decomposition by a Wiener filter where ϕ_{gx} and ϕ_{xx} are used in place of ϕ_{gs} and ϕ_{ss} respectively. The Wiener filter output proves that the above mentioned conditions are fulfilled in this case.

Conclusions

We have developed further design methods for signal decomposition by Wiener filters for noisy inputs and unknown wavelet shape.

It is possible to reduce the output noise by varying the width of the desired output pulse. However, when noise statistics is known, we have shown how to improve the Wiener filter performance by optimal selection of the noise weighting factor in the cost function. For the noisy situations the Wiener filter thus designed is particularly suitable since good resolution can be achieved by relatively short memory duration.

The extension of Wiener filters for noisy inputs and unknown wavelet shape is still open for further research.

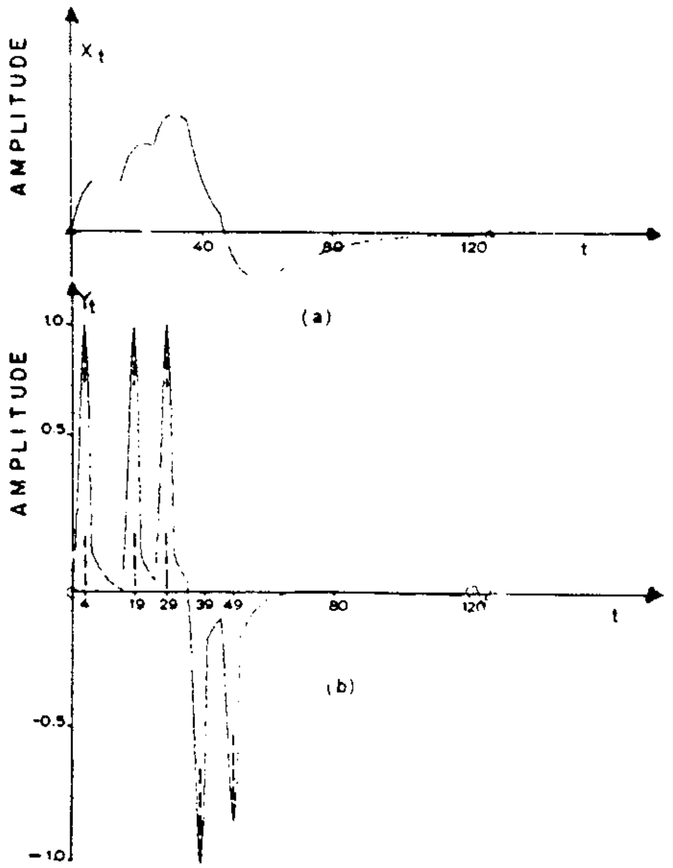

Figure 5

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